I-HOMOMORPHISM FOR BL-I-GENERAL L-FUZZY AUTOMATA

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Abstract. Taking into account the notion of BL-general fuzzy automaton, in the present study we define the notation of BL-intuitionistic general L-fuzzy automaton and I-bisimulation for BL-intuitionistic general L-fuzzy automaton. Then for a given BL-intuitionistic general L-fuzzy automaton, we obtain the greatest I-bisimulation. According to this notion, we give the structure of quotient BL-intuitionistic general L-fuzzy automaton. Fortunately, this quotient is the minimal BL-intuitionistic general L-fuzzy automaton. In addition, in this study, we show that if there is an I-bisimulation between two BL-intuitionistic general L-fuzzy automata, then they have the same behavior. Furthermore, we give an algorithm which determines the I-bisimulation between any two BL-intuitionistic general L-fuzzy automata. To clarify the notions and the results obtained in this paper, we have submitted some examples as well.

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Keywords: BL-general fuzzy automata; BL-intuitionistic general L-fuzzy automata; Bisimulation; Quotient automata; Minimal BL-general fuzzy automata

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1. Introduction

Fuzzy automata were introduced by W. G. Wee [42] in 1967 and Santos [36] in 1968. E.T. Lee and L.A. Zadeh [28] in 1969 gave the concept of fuzzy finite state automata. Fuzzy finite automata have offered many applications in several fields of science, for example in learning system, pattern recognition, neural networks, and database theory [20, 21, 30, 31, 33, 34, 43]. By adding non-membership value, Atanassov in 1986 [3] has extended the notion of fuzzy set to the intuitionistic fuzzy set (IFS), which may express more accurate and flexible information as compared with fuzzy sets. Intuitionistic fuzzy set (IFS) theory has many applications in several subjects, see [11, 12, 13, 14, 17, 24, 26, 27, 41, 23]. Recently, it has been found that it is highly useful to deal with vagueness. Gua and Buehrer [18] introduced the concept of vague sets, but Burillo [5] showed that the concept of vague sets was coincided with that of IFSs. These studies established a good foundation for the development of IFSs. These studies have established a good foundation for the development of IFSs, introducing the coincidence of IFSs with interval value fuzzy sets (IVFSs) [7].

Using the notion of intuitionistic fuzzy sets, W. L. Jun [25] introduced the notion of intuitionistic fuzzy finite state machines as a generalization of fuzzy finite state machines. According to the studies [25, 26], Zhang and Li [45] discussed intuitionistic fuzzy finite automata. K. Atanassov and S. Stoeva generalized the concept of IFS to intuitionistic L-fuzzy sets [4], where L is an appropriate lattice. A. Tepavcevic and T. Gerstenkorn gave a new definition of lattice-valued intuitionistic fuzzy sets in [19]. After that, on the basis of lattice-valued intuitionistic fuzzy sets, Yang et al. [44] presented the concepts of lattice-valued intuitionistic fuzzy finite state machines.

In 2004, M. Doostfatemeh and S.C. Kremer [15] extended the notion of fuzzy automata and gave the notion of general fuzzy automata. Their key motivation for introducing the notion general fuzzy automata was the insufficiency of the current literature to handle the applications which rely on fuzzy automata as a modeling tool, assigning membership values to active states of a fuzzy automaton, in order to resolve the multi-membership. Another important insufficiency of the current literature is the lack of methodologies which enable us to define and analyze the continuous operation of fuzzy automata. In 2014, M. Shamsizadeh and M. M.

Zahedi [37] gave the notion of max-min intuitionistic general fuzzy automata. In order to provide a general framework for formalizing statements of fuzzy nature, the concept of 'Basic Logic' (BL) has been suggested by Hájek [22]. Formulas of propositional BL may be interpreted by means of BL-algebras [40]. In 2012, Kh. Abolpour and M. M. Zahedi [1] extended the notion of general fuzzy automata and gave the notion of BL-general fuzzy automata (BL-GFA).

Bisimulations have been introduced by Milner [29] and Park [32]. Bisimulation has been widely used in many areas of computer science to model equivalence between various systems, and to reduce the number of states of these systems. The approach to bisimulations, proposed in [8, 9] for fuzzy automata, has been applied in ordinary nondeterministic automata and in weighted automata [10]. Today, they are employed in many areas of computer science, such as functional languages, object-oriented languages, databases, compiler optimizations, and verification tools [2, 6, 16, 35].

In 2015, M. Shamsizadeh and M.M Zahedi [39] defined the notion of bisimulation between two BL-general fuzzy automata. Taking into account the notions of the intuitionistic general fuzzy automaton, BL-general fuzzy automaton and bisimulation for BL-general fuzzy automaton, in the present study we define the notions BL-intuitionistic general L-fuzzy automaton and I-bisimulation between two BL-intuitionistic general L-fuzzy automata. This study mainly aims at obtaining the greatest I-bisimulation for a BL-intuitionistic general L-fuzzy automaton. Also, for a given BL-intuitionistic general L-fuzzy automaton, we realize the quotient BL-intuitionistic general L-fuzzy automaton such that this quotient is the minimal BL-intuitionistic general L-fuzzy automaton. Moreover, the authors show that if there is an I-bisimulation between two BL-intuitionistic general L-fuzzy automata, then there is a morphism between them. Finally, we give an algorithm, which determines the I-bisimulation between any two BL-intuitionistic general L-fuzzy automata and in the following we obtain its time complexity. Using the concept of IFS, all results of this study should be held for IVFSs, vague sets, and fuzzy sets.

2. Preliminaries

We review some definitions which are needed in the forthcoming sections.

Definition 1. [3] Let A in E is given. An intuitionistic fuzzy set (IFS) A^+ on E is an object of the following form

$$A^+ = \{\langle x, \mu_A(x), \nu_A(x)\rangle \big| x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the value of membership and the value of non-membership of element x in E to the set A, which is a subset of E, respectively. Also, for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2. [4] Let X be a nonempty set and L be a complete lattice with an involutive order reversing unary operation $N : L \to L$. An intuitionistic L-fuzzy set (ILFS) is an object of the form $A = \{(x, \mu(x), \nu(x)) | x \in E\}$, where μ and ν are functions $\mu : E \to L, \nu : E \to L$ in which for all $x \in X$, $\mu(x) \leq N(\nu(x))$.

In what follows in this paper, $L = (L, \leq_L, T, S, 0, 1)$ always denotes a bounded complete lattice, where endowed with an Lt-norm T, an Lt-conorm S, the least element 0 and the greatest element 1, also with an involutive order reversing unary operation $N : L \to L, \alpha, \beta \in L$ and $\alpha \leq N(\beta)$.

Definition 3. [38] An intuitionistic general L-fuzzy automaton (IGLFA) $\tilde{F}$ is a ten-tuple machine denoted by $\tilde{F} = (Q, X, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$, where

- Q is a set of states,
- X is a finite set of input symbols, $X = \{a_1, a_2, ..., a_m\}$,
- $\tilde{R}$ is the ILFS of start states, $\tilde{R} = \{(q, \mu^{t_0}(q), \nu^{t_0}(q)) \big| q \in R\}$, where R is a finite subset of Q,
- Z is a finite set of output symbols, $Z = \{b_1, b_2, ..., b_l\}$,
- $\tilde{\delta} : (Q \times L \times L) \times X \times Q \to L \times L$ is the augmented transition function,
- $\omega : Q \to Z$ is the output function,
- $F_1 = (F_1^T, F_1^S)$, where $F_1^T : L \times L \to L$ is a Lt-norm which is called the membership assignment function.

  Furthermore, $F_1^S : L \times L \to L$ is a Lt-conorm, where is the dual of F_1^T respect to involutive negation and it is called non-membership assignment function.
- $F_2 = (F_2^{TS}, F_2^{ST})$, where $F_2^{ST} : L^* \to L$ is a Lt-norm and is called the multi-non-membership function.

  Also, $F_2^{TS} : L^* \to L$ is a Lt-conorm, where it is the dual of F_3^{ST} respect to the involutive negation and it is called the multi-membership function.

Let $Q_{act}(t_i)$ be the set of all active states at time t_i for all $i \geq 0$. We have $Q_{act}(t_0) = \tilde{R}$ and $Q_{act}(t_i) = \{(q, \mu^{t_i}(q), \nu^{t_i}(q)) \big| \exists (q', \mu^{t_{i-1}}(q'), \nu^{t_{i-1}}(q')) \in Q_{act}(t_{i-1}), \exists a \in X, \delta(q', a, q) \in \Delta, \mu^{t_i}(q) >_L 0\}$ for all positive integer i.

Since $Q_{act}(t_i)$ is an ILFS, to show that a state q belongs to $Q_{act}(t_i)$, we write $q \in Domain(Q_{act}(t_i))$ and for simplicity of notation, we denote it by $q \in Q_{act}(t_i)$.

Definition 4. [22] A BL-algebra is algebra $(L, \wedge, \vee, *, \rightarrow, 0, 1)$ with four binary operations $\wedge, \vee, *, \rightarrow$ and two constants $0, 1$ such that: (i) $(L, \wedge, \vee, 0, 1)$ is a bounded lattice, (ii) $(L, *, 1)$ is a commutative monoid, (iii) $*$ and $\rightarrow$ form an adjoint pair, i.e., $x \leq y \rightarrow z$ if and only if $x * y \leq z$ for all $x, y, z \in L$, (iv) $x \wedge y = x * (x \rightarrow y)$, (v) $(x \rightarrow y) \vee (y \rightarrow x) = 1$.

Definition 5. [39] Let $\tilde{F} = (Q, X, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be a general fuzzy automaton and $\bar{Q} = (P(Q), \subseteq, \cap, \cup, \emptyset, Q)$ be a BL-algebra in Example 2 of [39]. Then the BL-general fuzzy automaton (BL-GFA) as a ten-tuple machine denoted by $\tilde{F}_l = (\bar{Q}, X, \tilde{R} = (\{q_0\}, \mu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$, where

(i) $\bar{Q} = P(Q)$, where Q is a finite set and $\bar{Q}$ is the powerset of Q,

(ii) X is a finite set of input symbols,

(iii) $\tilde{R}$ is the set of fuzzy start states,

(iv) $\bar{Z}$ is a finite set of output symbols, where $\bar{Z}$ is the power set of Z,

(v) $\omega_l : \bar{Q} \rightarrow \bar{Z}$ is the output function defined by: $\omega_l(Q_i) = \{\omega(q) \big| q \in Q_i\}$,

(vi) $\delta_l : \bar{Q} \times X \times \bar{Q} \rightarrow L$ is the transition function defined by: $\delta_l(\{p\}, a, \{q\}) = \delta(p, a, q)$ and $\delta_l(Q_i, a, Q_j) = \vee_{q_i \in Q_i, q_j \in Q_j} \delta(q_i, a, q_j)$, for all $Q_i, Q_j \in P(Q)$ and $a \in X$,

(vii) $f_l : \bar{Q} \times X \rightarrow \bar{Q}$ is the next state map defined by: $f_l(Q_i, a) = \cup_{q_i \in Q_i} \{q_j \big| \delta(q_i, a, q_j) \in \Delta\}$,

(viii) $\tilde{\delta}_l : (\bar{Q} \times L) \times X \times \bar{Q} \rightarrow L$ is the augmented transition function defined $\tilde{\delta}_l((Q_i, \mu^t(Q_i)), a, Q_j) = F_1(\mu^t(Q_i), \delta_l(Q_i, a, Q_j))$,

(ix) $F_1 : L \times L \rightarrow L$ is called membership assignment function,

(x) $F_2 : L^* \rightarrow L$ is called multi-membership resolution function.

Definition 6. [1] Let $\tilde{F}_l = (\bar{Q}, X, \tilde{R} = (\{q_0\}, \mu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$ be a BL-GFA. The run map of the BL-GFA $\tilde{F}_l$ is the map $\rho : X^* \rightarrow \bar{Q}$ is defined by the following induction: $\rho(\Lambda) = \{q_0\}$ and $\rho(a_1 a_2 ... a_n) = Q_{i_n}, \rho(a_1 a_2 ... a_n a_{n+1}) = f_l(Q_{i_n}, a_{n+1})$, where $(Q_{i_n}, \mu^{t_0+n}(Q_{i_n})) \in Q_{act}(a_1 a_2 ... a_n)$, for every $a_1, ..., a_n \in X$.

The map $\beta = \omega_l \circ \rho : X^* \to \bar{Z}$ is the behavior of $\tilde{F}_l$.

3. Intuitionistic BL-general L-fuzzy automata

This section is an attempt to introduce the concepts of BL-intuitionistic L-fuzzy automaton and I-bisimulation between two BL-intuitionistic L-fuzzy automata. We obtain the greatest I-bisimulation for the BL-intuitionistic general L-fuzzy automaton. Finally by taking into consideration the greatest I-bisimulation, we give the minimal BL-intuitionistic general L-fuzzy automaton.

Definition 7. Let $\tilde{F} = (Q, X, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2)$ be an intuitionistic general L-fuzzy automaton (IGLFA) and $\bar{Q} = (P(Q), \subseteq, \cap, \cup, \emptyset, Q)$ be a BL-algebra as in Example 1 of [39]. We define the BL-intuitionistic general L-fuzzy automaton (BL-IGLFA) as a ten-tuple machine denoted by $\widetilde{F_l} = (\bar{Q}, X, \widetilde{R} = (\{q_0\}, \mu^{t_0}(\{q_0\}, \nu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$, where

(i) $\bar{Q} = P(Q)$, where Q is a finite set and $\bar{Q}$ is the power set of Q,
(ii) X is a finite set of input symbols,
(iii) $\widetilde{R}$ is the set of fuzzy start states,
(iv) $\bar{Z}$ is a finite set of output symbols, where $\bar{Z}$ is the power set of Z,
(v) $\omega_l : \bar{Q} \to \bar{Z}$ is the output function defined by: $\omega_l(Q_i) = \{\omega(q) \big| q \in Q_i\}$,
(vi) $\delta_l : \bar{Q} \times X \times \bar{Q} \to L \times L$ is the intuitionistic transition function defined by:

$$\delta_l(\{p\}, a, \{q\}) = (\delta_{l\mu}(\{p\}, a, \{q\}), \delta_{l\nu}(\{p\}, a, \{q\})) = (\delta_\mu(p, a, q), \delta_\nu(p, a, q)),$$

also, we have $\delta_l(Q_i, a, Q_j) = (\delta_{l\mu}(Q_i, a, Q_j), \delta_{l\nu}(Q_i, a, Q_j))$, where

$$\delta_{l\mu}(Q_i, a, Q_j) = \vee\{\delta_{l\mu}(q_i, a, q_j) | q_i \in Q_i, q_j \in Q_j\},$$

and

$$\delta_{l\nu}(Q_i, a, Q_j) = \wedge\{\delta_{l\nu}(q_i, a, q_j) | q_i \in Q_i, q_j \in Q_j\},$$

for every $Q_i, Q_j \in \bar{Q}$ and $a \in X$,

(vii) $f_l : \bar{Q} \times X \to \bar{Q}$ is the next state map defined by: $f_l(Q_i, a) = \cup_{q_i \in Q_i}\{q_j \big| \delta(q_i, a, q_j) \in \Delta\}$,
(viii) $\tilde{\delta}_l : (\bar{Q} \times L \times L) \times X \times \bar{Q} \to L \times L$ is the augmented transition function,
(ix) $F_1 = (F_1^T, F_1^S)$, where $F_1^T : L \times L \to L$ is a L-tnorm which is called membership assignment function. Furthermore, $F_1^S : L \times L \to L$ is a L-tconorm, where is the dual of F_1^T respect to the involutive negation, which

is called non-membership assignment function. The process that takes place upon the transition from the state Q_i to Q_j on an input a is given by:

$$\tilde{\delta}_l((Q_i, \mu^t(Q_i), \nu^t(Q_i)), a, Q_j) = (\tilde{\delta}_{l\mu}((Q_i, \mu^t(Q_i), \nu^t(Q_i)), a, Q_j), \tilde{\delta}_{l\nu}((Q_i, \mu^t(Q_i), \nu^t(Q_i)), a, Q_j)),$$

where $\tilde{\delta}_{l\mu}((Q_i, \mu^t(Q_i), \nu^t(Q_i)), a, Q_j) = F_1^T(\mu^t(Q_i), \delta_{l\mu}(Q_i, a, Q_j))$, and

$$\tilde{\delta}_{l\nu}((Q_i, \mu^t(Q_i), \nu^t(Q_i)), a, Q_j) = F_1^S(\nu^t(Q_i), \delta_{l\nu}(Q_i, a, Q_j)),$$

(x) $F_2 = (F_2^{TS}, F_2^{ST})$, where $F_2^{ST} : L^* \to L$ is a L-tnorm which is called the multi-non-membership function. Also, $F_2^{TS} : L^* \to L$ is a L-tconorm, where it is the dual of F_3^{ST} respect to the involutive negation, it is called multi-membership function.

Example 1. Let $(L, \wedge, \vee, 0, 1)$ be a complete lattice as in Figure 1, where $N(0) = 1, N(1) = 0, N(a) = d, N(d) = a, N(b) = c, N(c) = b$, and $N(f) = e, N(e) = f$. Consider the intuitionistic general L-fuzzy automaton $\widetilde{F}_i = (Q_i, X, \tilde{\delta}^i, \widetilde{R}_i, Z, \omega_i, F_1, F_2)$,

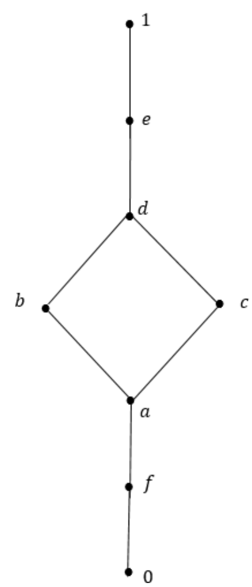

FIGURE 1. The complete lattice L of Example 3

$i = 1, 2$, where $Q_1 = \{q_1, q_2\}, Q_2 = \{p_1, p_2\}, \widetilde{R}_1 = \{(q_1, 1, 0)\}, \widetilde{R}_2 = \{(p_1, 1, 0)\}, X =$

$\{\sigma\}, Z = \{z_1, z_2\}, \omega_1(q_1) = \omega_1(q_2) = \omega_2(p_1) = \omega_2(p_2) = z_2$ and

$$\delta^1(q_1,\sigma,q_1) = (a,b),\ \delta^1(q_2,\sigma,q_1) = (b,a),$$
$$\delta^1(q_1,\sigma,q_2) = (c,a),\ \delta^1(q_2,\sigma,q_2) = (d,a),$$
$$\delta^2(p_1,\sigma,p_1) = (a,b),\ \delta^2(p_2,\sigma,p_1) = (b,a),$$
$$\delta^2(p_1,\sigma,p_2) = (c,a),\ \delta^2(p_2,\sigma,p_2) = (c,a).$$

By considering Definition 7, we have BL-IGLFAs $\widetilde{F}_{li}, i = 1,2$ as follows: $\widetilde{F}_{li} = (\bar{Q}_i, X, \tilde{R}_i, \bar{Z}, \omega_{li}, \delta^{li}, f_{li},$
$\tilde{\delta}^{li}, F_1, F_2), i = 1,2,$ where $\bar{Q}_1 = \{\emptyset, \{q_1\}, \{q_2\}, \{q_1,q_2\}\}$, $\bar{Q}_2 = \{\emptyset, \{p_1\}, \{p_2\}, \{p_1,p_2\}\}$, $\tilde{R}_1 = (\{q_1\},1,0)$, $\tilde{R}_2 = (\{p_1\},1,0)$, $\bar{Z} = \{\emptyset, \{z_1\}, \{z_2\}, \{z_1,z_2\}\}, \omega_{l1}(\{q_1\}) = \omega_{l1}(\{q_2\}) = \omega_{l1}(\{q_1,q_2\}) = \omega_{l2}(\{p_1\}) = \omega_{l2}(\{p_2\}) = \omega_{l2}(\{p_1,p_2\}) = \{z_2\}, f_{l1}(\{q_1\},\sigma) = f_{l1}(\{q_2\},\sigma) = f_{l1}(\{q_1,q_2\},\sigma) = \{q_1,q_2\}$, $f_{l2}(\{p_1\},\sigma) = f_{l2}(\{p_2\},\sigma) = f_{l2}(\{p_1,p_2\},\sigma) = \{p_1,p_2\}$ and

$$\delta^{l1}(\{q_1\},\sigma,\{q_1\}) = (a,b),\quad \delta^{l2}(\{p_1\},\sigma,\{p_1\}) = (a,b),$$
$$\delta^{l1}(\{q_1\},\sigma,\{q_2\}) = (c,a),\quad \delta^{l2}(\{p_1\},\sigma,\{p_2\}) = (c,a),$$
$$\delta^{l1}(\{q_1\},\sigma,\{q_1,q_2\}) = (c,a),\quad \delta^{l2}(\{p_1\},\sigma,\{p_1,p_2\}) = (c,a),$$
$$\delta^{l1}(\{q_2\},\sigma,\{q_1\}) = (b,a),\quad \delta^{l2}(\{p_2\},\sigma,\{p_1\}) = (b,a),$$
$$\delta^{l1}(\{q_2\},\sigma,\{q_2\}) = (d,a),\quad \delta^{l2}(\{p_2\},\sigma,\{p_2\}) = (c,a),$$
$$\delta^{l1}(\{q_2\},\sigma,\{q_1,q_2\}) = (d,a),\quad \delta^{l2}(\{p_2\},\sigma,\{p_1,p_2\}) = (d,a),$$
$$\delta^{l1}(\{q_1,q_2\},\sigma,\{q_1\}) = (b,a),\quad \delta^{l2}(\{p_1,p_2\},\sigma,\{p_1\}) = (b,a),$$
$$\delta^{l1}(\{q_1,q_2\},\sigma,\{q_2\}) = (d,a),\quad \delta^{l2}(\{p_1,p_2\},\sigma,\{p_2\}) = (c,a),$$
$$\delta^{l1}(\{q_1,q_2\},\sigma,\{q_1,q_2\}) = (d,a),\quad \delta^{l2}(\{p_1,p_2\},\sigma,\{p_1,p_2\}) = (d,a).$$

Definition 8. Let $(\bar{Q}, f_l, \delta_l)$ and $(\bar{Q}', f'_l, \delta'_l)$. Then $g : (\bar{Q}, f_l, \delta_l) \to (\bar{Q}', f'_l, \delta'_l)$ is called an I-homomorphism with threshold $\dfrac{\tau_1}{\tau_2}$ if there is a map of $\bar{Q}$ into $\bar{Q}'$ such that for every $Q_i, Q_j \in \bar{Q}$ the following hold:

(i) $g \circ f_l = f'_l \circ (g \times id_X)$,

(ii) $\tau_1 \le \delta_{l\mu}(f_l(Q_i,a_1),a_2,Q_j) \le \tau_2$ if and only if $\tau_1 \le \delta'_{l\mu}(g(f_l(Q_i,a_1)),a_2,g(Q_j)) \le \tau_2$,

(iii) if $\delta_{l\nu}(f_l(Q_i,a_1),a_2,Q_j) \le 1-\tau_2$, then $\delta'_{l\nu}(g(f_l(Q_i,a_1)),a_2,g(Q_j)) \le 1-\tau_2$,

(iv) if $\delta'_{l\nu}(g(f_l(Q_i, a_1)), a_2, g(Q_j)) \leq 1 - \tau_2$, then $\delta_{l\nu}(f_l(Q'_i, a_1), a_2, Q'_j) \leq 1 - \tau_2$, for some $Q'_i, Q'_j \in \bar{Q}$ such that $g(Q_i) = g(Q'_i)$ and $g(Q_j) = g(Q'_j)$.

We say that $g : (\bar{Q}, f_l, \delta_l) \to (\bar{Q}', f'_l, \delta'_l)$ is an I-homomorphism if and only if $g : (\bar{Q}, f_l, \delta_l) \to (\bar{Q}', f'_l, \delta'_l)$ is an I-homomorphism with threshold $\dfrac{0}{1}$.

Definition 9. Let $\tilde{F}_{li} = (\bar{Q}_i, X, \tilde{R}_i = (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2$ be two BL-IGLFAs. We say that $(g, g_{out}) : \tilde{F}_l \to \tilde{F}'_l$ is an I-morphism with threshold $\dfrac{\tau_1}{\tau_2}$ if and only if the following hold:

(i) $g(\{q_0\}) = \{q'_0\}$,
(ii) $g : (\bar{Q}, f_l, \delta_l) \to (\bar{Q}', f'_l, \delta'_l)$ is an I-homomorphism with threshold $\dfrac{\tau_1}{\tau_2}$,
(iii) $g_{out} \circ \omega_l = \omega'_l \circ g$.

We say that $(g, g_{out}) : \tilde{F}_l \to \tilde{F}'_l$ is an I-morphism if and only if $(g, g_{out}) : \tilde{F}_l \to \tilde{F}'_l$ is an I-morphism with threshold $\dfrac{0}{1}$.

Theorem 1. *For every I-morphism $(g, g_{out}) : \tilde{F}_l \to \tilde{F}'_l$ with threshold $\frac{\tau_1}{\tau_2}$ of BL-IGLFAs,*

(i) the run map ρ of $\tilde{F}_l$ is related to the run map ρ' of $\tilde{F}'_l$ by $\rho' = g \circ \rho$,
(ii) the behavior β of $\tilde{F}_l$ is related to the behavior of $\tilde{F}'_l$ by $\beta' = g_{out} \circ \beta$.

Proof. Let $\widetilde{F}_l = (\bar{Q}, X, \widetilde{R} = (\{q_0\},\ \mu^{t_0}(\{q_0\}, \nu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$ and $\widetilde{F}'_l = (\bar{Q}', X, \widetilde{R}' = (\{q'_0\}, \mu^{t_0}(\{q'_0\}, \nu^{t_0}(\{q'_0\})), \bar{Z}, \omega'_l, \delta'_l, f'_l, \tilde{\delta}'_l, F_1, F_2)$ be two BL-IGLFAs. Let ρ and ρ' be the run relation of $\tilde{F}_l$ and $\tilde{F}'_l$, respectively.

(i) We prove the claim by induction on $|x| = n$. Let $n = 0$. Then $x = \Lambda$. So $\rho(\Lambda) = \{q_0\}, \rho'(\Lambda) = \{q'_0\} = g(\{q_0\}) = g(\rho(\Lambda))$. If $n = 1$, then $x = a$. Therefore

$$\begin{aligned}\rho'(a) = f'_l(\{q'_0\}, a) &= f'_l(g(\{q_0\}), a)\\ &= g(f_l(\{q_0\}, a))\\ &= g(\rho(a)).\end{aligned}$$

Let the claim holds for any positive integer $n-1, n \geq 1$. Now, suppose that $|x| = n$ and $x = a_1a_2...a_n \in X^*$. So

$$\begin{aligned} \rho'(a_1...a_n) &= f_l'(\rho'(a_1...a_{n-1}), a_n) \\ &= f_l'(g(\rho(a_1...a_{n-1})), a_n) \\ &= g \circ f_l(\rho(a_1...a_{n-1}), a_n) \\ &= g(\rho(a_1...a_{n-1})). \end{aligned}$$

(ii) By considering $\rho' = g \circ \rho$, we have

$$\begin{aligned} \beta' = \omega_l' \circ \rho' = \omega_l' \circ (g \circ \rho) &= (\omega_l' \circ g) \circ \rho \\ &= (g_{out} \circ \omega_l) \circ \rho \\ &= g_{out} \circ (\omega_l \circ \rho) \\ &= g_{out} \circ \beta. \end{aligned}$$

Hence, the claim holds. □

Corollary 1. *Let $\tilde{F}_l$ and $\tilde{F}_l'$ be two BL-IGLFAs with the same output alphabet and let $(g, g_{out}) : \tilde{F}_l \to \tilde{F}_l'$ be an I-morphism with threshold $\frac{\tau_1}{\tau_2}$. Then $\tilde{F}_l$ and $\tilde{F}_l'$ have the same behavior.*

4. I-bisimulation for BL-intuitionistic general L-fuzzy automata

Definition 10. Let $\tilde{F}_{li} = (\bar{Q}_i, X, \tilde{R}_i = (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2$ be two BL-IGLFAs. Then the relation $\approx$ between $\bar{Q}_1$ and $\bar{Q}_2$ is called an intuitionistic bisimulation (I-bisimulation) between $\tilde{F}_{l1}$ and $\tilde{F}_{l2}$ if the following hold:

(1) $\{q_{01}\} \approx \{q_{02}\}$,

(2) $Q' \approx Q''$ implies that

$$(\forall \alpha \in L)(Q_1' \in \bar{Q}_1)(a \in X)(\delta_{l\mu 1}(Q', a, Q_1') = \alpha)$$
$$\Longrightarrow ((\exists Q_2' \in \bar{Q}_2)\delta_{l\mu 2}(Q'', a, Q_2') \geq \alpha, Q_1' \approx Q_2') \text{ and vice versa,}$$

(3) $Q' \approx Q''$ implies that

$$(\forall \beta \in L)(Q_1' \in \bar{Q}_1)(a \in X)(\delta_{l\nu 1}(Q', a, Q_1') = \beta)$$
$$\Longrightarrow (\exists Q_2' \in \bar{Q}_2)\delta_{l\nu 2}(Q'', a, Q_2') \leq \beta, Q_1' \approx Q_2') \text{ and vice versa,}$$

(4) $Q' \approx Q''$ implies that $\omega_{l1}(Q') = \omega_{l2}(Q'')$,

where $Q' \in \bar{Q}_1$ and $Q'' \in \bar{Q}_2$.

Definition 11. Let $\widetilde{F}_l = (\bar{Q}, X, \widetilde{R} = (\{q_0\}, \mu^{t_0}(\{q_0\}), \nu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$ be a BL-IGLFA. Then $\tilde{F}_l$ is a minimal BL-IGLFA if for every BL-IGLFA $\widetilde{F}'_l$, which $\widetilde{F}'_l$ is I-bisimilar to $\widetilde{F}_l$, $|\widetilde{F}_l| \leq |\widetilde{F}'_l|$.

Note 1. Let $\widetilde{F}_{li} = (\bar{Q}_i, X, \widetilde{R}_i = (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2)$, $i = 1, 2, 3$ be three BL-IGLFA.
(i) Let $\approx$ be an I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$. Clearly, its reverse is an I-bisimulation between $\widetilde{F}_{l2}$ and $\widetilde{F}_{l1}$. So, the relation $\approx$ is a symmetric relation between $\bar{Q}_1$ and $\bar{Q}_2$.
(ii) Let $\approx_1$ be an I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$ and $\approx_2$ be an I-bisimulation between $\widetilde{F}_{l2}$ and $\widetilde{F}_{l3}$. Then their composition as follow:

$$\approx = \approx_1 \circ \approx_2 = \{(P, P') \big| \exists Q' \in \bar{Q}_2, P \approx_1 Q' and\ Q' \approx_2 P'\},$$

is an I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l3}$. So, we proof the claim as follows:

(1) We have $\{q_{01}\} \approx_1 \{q_{02}\}$ and $\{q_{02}\} \approx_2 \{q_{03}\}$. Then $\{q_{01}\} \approx \{q_{03}\}$.

(2) Let $Q'_1 \approx Q'_3$. Then there is $Q'_2 \in \bar{Q}_2$ such that $Q'_1 \approx_1 Q'_2$ and $Q'_2 \approx_2 Q'_3$. Also, by considering Definition 10, and $Q'_1 \approx_1 Q'_2$ we have

$$(\forall \alpha \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\mu 1}(Q'_1, a, Q''_1) = \alpha)$$
$$\Longrightarrow (\exists Q''_2 \in \bar{Q}_2)(\delta_{l\mu 2}(Q'_2, a, Q''_2) \geq \alpha, Q''_1 \approx_1 Q''_2) \text{ and vice versa.}$$

Also, $Q'_2 \approx_2 Q'_3$ implies that

$$(\forall \alpha \in L)(Q''_2 \in \bar{Q}_2)(a \in X)(\delta_{l\mu 2}(Q'_2, a, Q''_2) = \alpha)$$
$$\Longrightarrow (\exists Q''_3 \in \bar{Q}_3)(\delta_{l\mu 3}(Q'_3, a, Q''_3) \geq \alpha, Q''_2 \approx_2 Q''_3) \text{ and vice versa.}$$

So, obviously

$$(\forall \alpha \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\mu 1}(Q'_1, a, Q''_1) = \alpha)$$
$$\Longrightarrow (\exists Q''_3 \in \bar{Q}_3)(\delta_{l\mu 3}(Q'_3, a, Q''_3) \geq \alpha, Q''_1 \approx Q''_3) \text{ and vice versa.}$$

(3) The proof is similar to (2).

(4) Let $Q'_1 \approx Q'_3$. Then there is $Q'_2 \in \bar{Q}_2$ such that $Q'_1 \approx_1 Q'_2$ and $Q'_2 \approx_2 Q'_3$. So, $\omega_{l1}(Q'_1) = \omega_{l2}(Q'_2) = \omega_{l3}(Q'_3)$. Hence, the relation $\approx$ is a transitive relation between $\bar{Q}_1$ and $\bar{Q}_3$.

Lemma 1. *Let* $\widetilde{F}_{li} = (\bar{Q}_i, X, (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2)$, $i = 1, 2$ *be two BL-IGLFAs and let* $\{\approx_i \big| i \in l\}$ *be an arbitrary nonempty set of I-bisimulations between* $\widetilde{F}_{l1}$ *and* $\widetilde{F}_{l2}$. *Then* $\approx = \cup_{i\in l} \approx_i$ *is an I-bisimulation between* $\widetilde{F}_{l1}$ *and* $\widetilde{F}_{l2}$.

Proof. Let $\approx = \cup_{i\in l} \approx_i$. Then $Q' \approx Q''$ if and only if there is $i \in l$ such that $Q' \approx_i Q''$. For every $i \in l$, we have $\{q_{01}\} \approx_i \{q_{02}\}$. So, $\{q_{01}\} \approx \{q_{02}\}$. Let $Q'_1 \approx Q'_2$. Then $Q'_1 \approx_i Q'_2$, for some $i \in l$. Therefore,

$$(\forall \alpha \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\mu 1}(Q'_1, a, Q''_1) = \alpha) \Longrightarrow (\exists Q''_2 \in \bar{Q}_2)(\delta_{l\mu 2}(Q'_2, a, Q''_2) \geq \alpha, Q''_1 \approx_i Q''_2) \text{ and vice versa.}$$

So,

$$(\forall \alpha \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\mu 1}(Q'_1, a, Q''_1) = \alpha) \Longrightarrow (\exists Q''_2 \in \bar{Q}_2)(\delta_{l\mu 2}(Q'_2, a, Q''_2) \geq \alpha, Q''_1 \approx Q''_2) \text{ and vice versa.}$$

In a similar way, if

$$(\forall \beta \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\nu 1}(Q'_1, a, Q''_1) = \beta) \Longrightarrow (\exists Q''_2 \in \bar{Q}_2)(\delta_{l\nu 2}(Q'_2, a, Q''_2) \leq \beta, Q''_1 \approx_i Q''_2) \text{ and vice versa,}$$

then,

$$(\forall \beta \in L)(Q''_1 \in \bar{Q}_1)(a \in X)(\delta_{l\nu 1}(Q'_1, a, Q''_1) = \beta) \Longrightarrow (\exists Q''_2 \in \bar{Q}_2)(\delta_{l\nu 2}(Q'_2, a, Q''_2) \leq \beta, Q''_1 \approx Q''_2) \text{ and vice versa.}$$

Finally, if $Q_1 \approx Q_2$, then there exists $i \in l$ such that $Q_1 \approx_i Q_2$ and $\omega_{l1}(Q_1) = \omega_{l2}(Q_2)$. Hence, the claim holds. □

Theorem 2. *Let* $\widetilde{F}_{li} = (\bar{Q}_i, X, \widetilde{R}_i = (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2$ *be two BL-IGLFAs and let* $\approx$ *be an I-bisimulation between* $\widetilde{F}_{l1}$ *and* $\widetilde{F}_{l2}$. *Then* $\beta_{\tilde{F}_{l1}} = \beta_{\tilde{F}_{l2}}$.

Proof. Let $\approx$ be an I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$ and let ρ_1 and ρ_2 be the run relations of $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$, respectively. We show that for every $a_1...a_n = x \in X^*$ there exist $Q'_1 \in \bar{Q}_1$ and $Q'_2 \in \bar{Q}_2$ such that $\rho_1(x) \approx Q'_2 \subseteq f_{l2}(\rho_2(a_1a_2...a_{n-1}), a_n)$ and $\rho_2(x) \approx Q'_1 \subseteq f_{l1}(\rho_1(a_1a_2...a_{n-1}), a_n)$. We prove the claim by induction on $|x| = n$. Now, let $|x| = 0$. Then $x = \Lambda$ and $\rho_1(\Lambda) = \{q_{01}\} \approx \{q_{02}\} = \rho_2(\Lambda)$. Let

$x = a \in X$. Then $\rho_1(a) = f_{l1}(\{q_{01}\}, a)$. Suppose that $\alpha, \beta \in L, \alpha \leq N(\beta)$ be such that $\delta_{l\mu 1}(\{q_{01}\}, a, f_{l1}(\{q_{01}\}, a)) = \alpha$ and $\delta_{l\nu 1}(\{q_{01}\}, a, f_{l1}(\{q_{01}\}, a)) = \beta$. Then by considering Definition 10, there exist $Q_2', Q_2'' \in \bar{Q}_2$ such that $\delta_{l\mu 2}(\{q_{02}\}, a, Q_2') \geq \alpha, \delta_{l\nu 2}(\{q_{02}\}, a, Q_2'') \leq \beta$, $\rho_1(a) \approx Q_2' \subseteq f_{l2}(\{q_{02}\}, a)$ and $\rho_1(a) \approx Q_2'' \subseteq f_{l2}(\{q_{02}\}, a)$. Now, let the claim holds for every $y \in X^*$ such that $|y| = n-1, n > 0$. Let $a_1a_2...a_n = x \in X^*, \alpha, \beta \in L, P_2' \approx \rho_1(a_1...a_{n-1}) \approx Q_2'$ and $\delta_{l\mu 1}(\rho_1(a_1...a_{n-1}), a_n, \rho_1(a_1...a_n)) = \alpha$ and $\delta_{l\nu 1}(\rho_1(a_1...a_{n-1}), a_n, \rho_1(a_1...a_n)) = \beta$. Then there exist $Q_2'', P_2'' \in \bar{Q}_2$ such that $\delta_{l\mu 2}(Q_2', a_n, Q_2'') \geq \alpha$, $\delta_{l\nu 2}(P_2', a_n, P_2'') \leq \beta$ and $\rho_1(x) \approx Q_2'' \subseteq f_{l2}(Q_2', a_n) \subseteq f_{l2}(\rho_2(a_1...a_{n-1}), a_n)$, where $P_2' \approx f_{l1}(\rho_1(a_1...a_{n-1}), a_n) \approx Q_2'$. So,

$$\rho_1(x) \approx P_2'' \subseteq f_{l2}(P_2', a_n) \subseteq f_{l2}(\rho_2(a_1...a_{n-1}), a_n).$$

Similarly, there is $Q_1' \in \bar{Q}_1$ such that $\rho_2(x) \approx Q_1' \subseteq f_{l1}(\rho_1(a_1...a_{n-1}), a_n)$. Therefore, for every $a_1a_2...a_n = x \in X^*$

$$\beta_{\tilde{F}_{l1}}(x) = \omega_{l1}(\rho_1(x)) = \omega_{l2}(Q_2') \subseteq \omega_{l2}(f_{l2}(\rho_2(a_1...a_{n-1}), a_n)) = \omega_{l2}(\rho_2(x)) = \beta_{\tilde{F}_{l2}}(x),$$

for some $Q_2' \in \bar{Q}_2$ such that $\rho_1(x) \approx Q_2'$. Also,

$$\beta_{\tilde{F}_{l2}}(x) = \omega_{l2}(\rho_2(x)) = \omega_{l1}(Q_1') \subseteq \omega_{l1}(f_{l1}(\rho_1(a_1...a_{n-1}), a_n)) = \omega_{l1}(\rho_1(x)) = \beta_{\tilde{F}_{l1}}(x),$$

for some $Q_1' \in \bar{Q}_1$ such that $\rho_2(x) \approx Q_1'$. Hence, $\beta_{\tilde{F}_{l1}}(x) = \beta_{\tilde{F}_{l2}}(x)$.

□

Notice that the converse of Theorem 2, does not hold, i.e., if $\widetilde{F}_{li} = (\bar{Q}_i, X, \widetilde{R}_i, \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2$, be two BL-IGLFAs and $\beta_{\tilde{F}_{l1}}(x) = \beta_{\tilde{F}_{l2}}(x)$, then it is not necessary that there exists a I-bisimulation between them.

Example 2. Let BL-IGLFA $\widetilde{F}_{l1}$ be as Example 1, also, consider BL-IGLFA $\widetilde{F}_{l2}$ as follows:

$$\widetilde{F}_{l2} = (\bar{Q}_2, X, \tilde{R}_2, \bar{Z}, \omega_{l2}, \delta^{l2}, f_{l2}, \tilde{\delta}^{l2}, F_1, F_2),$$

where $\bar{Q}_2 = \{\emptyset, \{p_1\}, \{p_2\}, \{p_1, p_2\}\}$, $\bar{Z} = \{\emptyset, \{z_1\}, \{z_2\}, \{z_1, z_2\}\}, \omega_{l2}(\{p_1\}) = \omega_{l2}(\{p_2\}) = \omega_{l2}(\{p_1, p_2\})$, $f_{l2}(\{p_1\}, \sigma) = f_{l2}(\{p_2\}, \sigma) = f_{l2}(\{p_1, p_2\}, \sigma) = \{p_1, p_2\}$, and

$$\begin{aligned}
&\delta^{l2}(\{p_1\}, \sigma, \{p_1\}) = (0, 1), \quad \delta^{l2}(\{p_1\}, \sigma, \{p_2\}) = (a, 1),\\
&\delta^{l2}(\{p_1\}, \sigma, \{p_1, p_2\}) = (0, 1), \quad \delta^{l2}(\{p_2\}, \sigma, \{p_1\}) = (a, 1),\\
&\delta^{l2}(\{p_2\}, \sigma, \{p_2\}) = (0, 1), \quad \delta^{l2}(\{p_2\}, \sigma, \{p_2\}) = (a, 1),\\
&\delta^{l2}(\{p_2\}, \sigma, \{p_1, p_2\}) = (0, 1), \quad \delta^{l2}(\{p_1, p_2\}, \sigma, \{p_1\}) = (a, 1),\\
&\delta^{l2}(\{p_1, p_2\}, \sigma, \{p_2\}) = (0, 1), \quad \delta^{l2}(\{p_1, p_2\}, \sigma, \{p_1, p_2\}) = (a, 1).
\end{aligned}$$

It is clear that $\beta_{\tilde{F}_{l1}}(x) = \beta_{\tilde{F}_{l2}}(x)$, but there is not any I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$.

Definition 12. Let $\widetilde{F}_{li} = (\bar{Q}_i, X, (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2$ be two BL-IGLFAs and $\approx$ be an I-bisimulation between $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$. Then the support of $\approx$ in $\widetilde{F}_{l1}$ is the set $C_{\approx}(\bar{Q}_2)$, the set of states of $\widetilde{F}_{l1}$ that are related by $\approx$ to some states of $\widetilde{F}_{l2}$.

Definition 13. Let $\widetilde{F}_l$ be a BL-IGLFA. We say that $\emptyset \neq Q' \in \bar{Q}$ is an accessible state if there exists $x \in X^*$ such that $f_l(\{q_0\}, x) = Q'$.

Note that an I-bisimulation between a BL-IGLFA and itself is called an I-bisimulation on BL-IGLFA.

Theorem 3. *Let $\widetilde{F}$ be a BL-IGLFA and let B be the set of all I-bisimulations on $\widetilde{F}$. Then union of all the relations in B is an I-bisimulation on $\widetilde{F}_l$ and also it is an equivalence relation on $\bar{Q}$.*

Proof. The proof is similar to the proof of Theorem 2 [39]. □

Let $\equiv$ be the union of all I-bisimulations on $\widetilde{F}_l$. We define $[P] = \{Q' \big| P \equiv Q'\}$, $\simeq = \{(P, [P]) \big| P \in \bar{Q}\}$, and $A' = \{[P] \big| P \in A\}$, for any $A \subseteq \bar{Q}$.

Lemma 2. *For every $A, B \subseteq \bar{Q}$:*

- *(i)* $A \subseteq C_{\equiv}(B)$ *if and only if* $A' \subseteq B'$,
- *(ii)* $A \equiv B$ *if and only if* $A' = B'$,
- *(iii)* $A \simeq A'$.

Proof. The proof is similar to the proof of Lemma 2, of [39]. □

Definition 14. Let $\widetilde{F}_l = (\bar{Q}_l, X, (\{q_0\}, \mu^{t_0}(\{q_0\}), \nu^{t_0}(\{q_0\})), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$ be a BL-IGLFA and let $\equiv$ be the union of all I-bisimulations on $\widetilde{F}_l$. Define the quotient BL-IGLFA $\widetilde{F}'_l$ as follows: $\widetilde{F'}_l = (\bar{Q'}_l, X, \widetilde{R'}, \bar{Z}, \omega'_l, \delta'_l, f'_l, \tilde{\delta'}_l, F_1, F_2)$, where $\bar{Q}'_l = \{[Q'] \big| Q' \in \bar{Q}_l\}, [Q'] = \{P \big| Q' \equiv P\}, \widetilde{R'} = [\{q_0\}], \mu^{t_0}([\{q_0\}]) = \mu^{t_0}(\{q_0\}), \nu^{t_0}([\{q_0\}]) = \nu^{t_0}(\{q_0\})$, $f'_l : \bar{Q'}_l \times X \to \bar{Q'}_l$ by $f'_l([Q'], a) = [f_l(Q', a)]$, $\delta'_l : \bar{Q'}_l \times X \times \bar{Q'}_l \to L \times L$ by $\delta'_l([Q'], a, [P]) = (\delta'_{l\mu}([Q'], a, [P]), \delta'_{l\nu}([Q'], a, [P]))$, where

(1)
$$\delta'_{l\mu}([Q'], a, [P]) = \vee\{\delta_{l\mu}(Q'', a, P') \big| Q'' \equiv Q', P' \equiv P\} = \vee\{\delta_{l\mu}(Q', a, P') \big| P' \equiv P\},$$

(2)
$$\delta'_{l\nu}([Q'], a, [P]) = \wedge\{\delta_{l\nu}(Q'', a, P') \big| Q'' \equiv Q', P' \equiv P\} = \wedge\{\delta_{l\nu}(Q', a, P') \big| P' \equiv P\},$$

and $\omega'_l : \bar{Q'}_l \to Z$ by $\omega'_l([Q']) = \omega_l(Q')$.

Clearly, $\tilde{\delta}_{l\mu}$ and $\tilde{\delta}_{l\nu}$ are well-defined, so $\tilde{\delta}_l$ is well-defined. Obviously, ω'_l is well-defined.

Theorem 4. *Let $\widetilde{F}_l$ be a BL-IGLFA with no inaccessible states and $\equiv$ be the greatest I-bisimulation on $\bar{Q}_l$. The quotient BL-IGLFA $\widetilde{F'}_l$ on $\widetilde{F}_l$, under I-bisimulation $\equiv$, is an I-morphism to $\widetilde{F}_l$.*

Proof. By considering Theorem 3 of [39] the proof is clear. □

Theorem 5. *Let $\widetilde{F}_l$ be a BL-IGLFA with no inaccessible states and let $\widetilde{F}'_l$ be the quotient BL-IGLFA of $\widetilde{F}_l$. Then $\widetilde{F}_l$ and $\widetilde{F}'_l$ have the same behavior.*

Proof. Clearly, by considering Theorems 1, 4, and Corollary 1. □

Theorem 6. *The relation $\simeq = \{(P, [P]) \big| P \in \bar{Q}_l\}$ is an I-bisimulation between BL-IGLFA $\widetilde{F}_l$ and quotient BL-IGLFA $\widetilde{F'}_l$, where $[P] = \{Q' \big| P \equiv Q'\}$. So, $\widetilde{F}_l$ and $\widetilde{F'}_l$ have the same behavior.*

Proof. Clearly, $\{q_0\} \simeq [\{q_0\}]$. Let $P \simeq [Q']$. If there are $P' \in \bar{Q}_l$ and $a \in X$ such that $\delta_{l\mu}(P, a, P') = \alpha$, then there exists $Q'' \in \bar{Q}'_l$ such that $\delta_{l\mu}(Q', a, Q'') \geq \alpha$ and $P' \equiv Q''$. So, $\delta'_{l\mu}([Q'], a, [Q'']) \geq \alpha$ and $P' \simeq [Q'']$. Now, let there are $[Q''] \in \bar{Q}'_l$ and $a \in X$ such that $\delta'_{l\mu}([Q'], a, [Q'']) = \alpha$. Then there is $S \equiv Q''$ such that $\delta'_{l\mu}([Q'], a, [Q'']) = \delta_{l\mu}(Q', a, S) = \alpha$. Therefore, there exists $P' \in \bar{Q}_l$ such that $\delta_{l\mu}(P, a, P') \geq \alpha$ and $P' \equiv S$ so $P' \simeq [Q'']$.

Also, if there are $P' \in \bar{Q}_l$ and $a \in X$ such that $\delta_{l\nu}(P, a, P') = \beta$, then there exists $Q'' \in \bar{Q}'_l$ such that $\delta_{l\nu}(Q', a, Q'') \leq \beta$ and $P' \equiv Q''$. Therefore, $\delta'_{l\nu}([Q'], a, [Q'']) \leq \beta$ and $P' \simeq [Q'']$. Let there are $[Q''] \in \bar{Q}'_l$ and $a \in X$ such that $\delta'_{l\nu}([Q'], a, [Q'']) = \beta$. Then there is $P' \equiv \bar{Q}_l$ such that $\delta_{l\nu}(P, a, P') \leq \beta' \leq \beta$, and $P' \simeq [Q'']$.

Now, let $P \simeq [Q']$. Then $P \equiv Q'$ and $\omega_l(P) = \omega_l(Q') = \omega'_l([Q'])$. Therefore $\widetilde{F}_l$ and $\widetilde{F'}_l$ are I-bisimulation. So, by considering Theorem 2, $\widetilde{F}_l$ and $\widetilde{F'}_l$ have the same behavior. □

Lemma 3. *The only I-bisimulation on the quotient BL-IGLFA $\widetilde{F'}_l$ is the identity relation.*

Proof. The proof is similar to the proof of Lemma 3 of [39]. □

Two BL-IGLFAs $\widetilde{F}_{l1}$ and $\widetilde{F}_{l2}$ are called I-bisimilar if there exists an I-bisimulation between them.

Theorem 7. *Let $\widetilde{F}_{l1}$ be a BL-IGLFA with no inaccessible states and $\equiv$ be the greatest I-bisimulation on $\bar{Q}_{l1}$. Then the quotient BL-IGLFA $\widetilde{F'}_{l1}$ is the minimal BL-IGLFA I-bisimilar to $\widetilde{F}_{l1}$.*

Proof. The proof is similar to the proof of Theorem 6 [39]. □

The following algorithm, for two given BL-IGLFAs determines an I-bisimulation between them. Also, if there is no I-bisimulation between them the algorithm stops and reports failure.

1. Algorithm for computing I-bisimulation

Step 1. **input:** Two BL-IGLFAs $\widetilde{F}_{li} = (\bar{Q}_i, X, (\{q_{0i}\}, \mu^{t_0}(\{q_{0i}\}), \nu^{t_0}(\{q_{0i}\})), \bar{Z}, \omega_{li}, \delta_{li}, f_{li}, \tilde{\delta}_{li}, F_1, F_2), i = 1, 2, Q' \in \bar{Q}_1, Q'' \in \bar{Q}_2, X = \{a_1, a_2, ..., a_n\}, j = 1,$

Step 2. $Q' \approx' Q''$ if and only if $\omega_{l1}(Q') = \omega_{l2}(Q'')$,

Step 3. If $\{q_{01}\} \approx' \{q_{02}\}$, then assume $\{q_{01}\} \approx_j \{q_{02}\}$. Also, let $Q' \approx_j Q''$ if and only if $Q' \approx' Q''$, where $Q' \neq \{q_{01}\}$ and $Q'' \neq \{q_{02}\}$,

Step 4. If $\{q_{01}\} \approx_j \{q_{02}\}$, then assume $k = 1$, $j = j + 1$, else go to Step 9,

Step 5. $Q' \approx_j Q''$ if and only if $Q' \approx_{j-1} Q''$ and

$$(\forall \alpha \in L)(Q'_1 \in \bar{Q}_1)(a_k \in X)(\delta_{l\mu 1}(Q', a_k, Q'_1) = \alpha)$$
$$\Longrightarrow (\exists Q'_2 \in \bar{Q}_2)(\delta_{l\mu 2}(Q'', a_k, Q'_2) \geq \alpha, Q'_1 \approx_{j-1} Q'_2) \text{ and vice versa,}$$

and

$$(\forall \beta \in L)(Q_1' \in \bar{Q}_1)(a_k \in X)(\delta_{l\nu 1}(Q', a_k, Q_1') = \beta)$$
$$\Longrightarrow (\exists Q_2' \in \bar{Q}_2)(\delta_{l\nu 2}(Q'', a_k, Q_2') \leq \beta, Q_1' \approx_{j-1} Q_2') \text{ and vice versa,}$$

Step 6. $k = k + 1$, if $k > n$, then go to the next step, else go to Step 5,

Step 7. if $\approx_j = \approx_{j-1}$, then go to the next step, else go to Step 4,

Step 8. **output:** $\approx = \approx_j$,

Step 9. **output:** fail.

Steps 3 to 5 of The algorithm, are a loop. This loop must be repeated at most $\max\{|\bar{Q}_1|, |\bar{Q}_2|\} + 1$ times. So, by considering $|X|$ and Steps 3 to 5, the order of time complexity is at most $O(|X||\bar{Q}_1||\bar{Q}_2|(\max\{|\bar{Q}_1|, |\bar{Q}_2|\}^2))$.

Example 3. Let $(L, \wedge, \vee, 0, 1)$ be a complete lattice as in Figure 1 and BL-IGLFA as Example 1. Then we have:

Stage 1.
1. $j = 1, X = \{\sigma\}, k = 1$,
2. $\{q_1\} \approx' \{p_1\} \approx' \{q_2\} \approx' \{p_2\} \approx' \{q_1, q_2\} \approx' \{p_1, p_2\}$,
3. $\{q_2\} \approx_1 \{p_2\} \approx_1 \{q_1, q_2\} \approx_1 \{p_1, p_2\}$, $\{q_1\} \approx_1 \{p_1\}$
4. $k = 1, j = 2$,
5. $\{q_1\} \approx_2 \{p_1\}$, $\{q_2\} \approx_2 \{p_2\} \approx_2 \{p_1, p_2\} \approx_2 \{q_1, q_2\}$,
6. $k = 2$,
7. $\approx_2 = \approx_1$,
8. Output: $\approx = \approx_2$.

Example 4. Let $(L, \wedge, \vee, 0, 1)$ be a complete lattice as in Figure 1. Consider the intuitionistic general L-fuzzy automaton $\widetilde{F} = (Q, X, \tilde{\delta}, \widetilde{R}, Z, \omega, F_1, F_2)$, where $Q = \{q_0, q_1\}$, $\widetilde{R} = \{(q_0, 1, 0)\}, X = \{\sigma_1, \sigma_2\}, Z = \{z_1, z_2\}, \omega(q_0) = \omega(q_1) = z_1$ and

$$\delta(q_0, \sigma_1, q_0) = (a, 0),\ \delta(q_0, \sigma_1, q_1) = (b, c),$$
$$\delta(q_1, \sigma_1, q_0) = (d, 0),\ \delta(q_1, \sigma_1, q_1) = (b, c),$$
$$\delta(q_0, \sigma_2, q_1) = (c, b),\ \delta(q_1, \sigma_2, q_1) = (1, 0).$$

By considering Definition 7, we have BL-IGLFA as follows: $\widetilde{F}_l = (\bar{Q}, X, (\{q_0\}, 1, 0), \bar{Z}, \omega_l, \delta_l, f_l, \tilde{\delta}_l, F_1, F_2)$, where $\bar{Q} = \{\emptyset, \{q_0\}, \{q_1\}, \{q_0, q_1\}\}$, $\bar{Z} = \{\emptyset, \{z_1\}, \{z_2\}, \{z_1, z_2\}\}$, $\omega_l(\{q_0\}) = \omega_l(\{q_1\}) = \omega_l(\{q_0, q_1\}) = \{z_1\}$, $f_l(\{q_0\}, \sigma_1) = f_l(\{q_1\}, \sigma_1) = f_l(\{q_0, q_1\}, \sigma_1) =$

$\{q_0, q_1\}, f_l(\{q_0\}, \sigma_2) = f_l(\{q_1\}, \sigma_2) = f_l(\{q_0, q_1\}, \sigma_2) = \{q_1\}$ and

$$\delta_l(\{q_0\}, \sigma_1, \{q_0\}) = (a, 0), \quad \delta_l(\{q_0\}, \sigma_1, \{q_1\}) = (b, c),$$
$$\delta_l(\{q_0\}, \sigma_1, \{q_0, q_1\}) = (b, 0), \quad \delta_l(\{q_1\}, \sigma_1, \{q_0\}) = (d, 0),$$
$$\delta_l(\{q_1\}, \sigma_1, \{q_1\}) = (b, c), \quad \delta_l(\{q_1\}, \sigma_1, \{q_0, q_1\}) = (d, 0),$$
$$\delta_l(\{q_0, q_1\}, \sigma_1, \{q_0\}) = (d, 0), \quad \delta_l(\{q_0, q_1\}, \sigma_1, \{q_1\}) = (b, c),$$
$$\delta_l(\{q_0, q_1\}, \sigma_1, \{q_0, q_1\}) = (d, 0) \ \delta_l(\{q_0\}, \sigma_2, \{q_1\}) = (c, d),$$
$$\delta_l(\{q_0\}, \sigma_2, \{q_0, q_1\}) = (c, d), \quad \delta_l(\{q_1\}, \sigma_2, \{q_1\}) = (1, 0),$$
$$\delta_l(\{q_1\}, \sigma_2, \{q_0, q_1\}) = (1, 0), \quad \delta_l(\{q_0, q_1\}, \sigma_2, \{q_1\}) = (1, 0),$$
$$\delta_l(\{q_0, q_1\}, \sigma_2, \{q_0, q_1\}) = (1, 0).$$

By taking into account the Definition 10, $[\{q_1\}] = [\{q_0, q_1\}]$, so, we have the quotient BL-intuitionistic general L-fuzzy automaton of $\widetilde{F}_l$, which is called $\widetilde{F}'_l$, as follows: $\bar{Q}' = \{[\emptyset], [\{q_0\}], [\{q_1\}]\}, \bar{Z} = \{\emptyset, \{z_1\}, \{z_2\}, \{z_1, z_2\}\}, \omega'_l([\{q_0\}]) = \omega'_l([\{q_1\}]) = \{z_1\}$ and

$$\delta'_l([\{q_0\}], \sigma_1, [\{q_0\}]) = (a, 0), \ \delta'_l([\{q_0\}], \sigma_1, [\{q_1\}]) = (b, 0),$$
$$\delta'_l([\{q_1\}], \sigma_1, [\{q_0\}]) = (d, 0), \ \delta'_l([\{q_1\}], \sigma_1, [\{q_1\}]) = (d, 0),$$
$$\delta'_l([\{q_0\}], \sigma_2, [\{q_1\}]) = (c, d), \ \delta'_l([\{q_1\}], \sigma_2, [\{q_1\}]) = (1, 0).$$

Clearly, $\simeq = \{(P, [P]) \big| P \in \bar{Q}\}$ is an I-bisimulation between $\widetilde{F}_l$ and $\widetilde{F}'_l$, where $\{q_0\} \simeq [\{q_0\}], \{q_1\} \simeq [\{q_1\}], \{q_0, q_1\} \simeq [\{q_1\}]$. Now, define $g : \bar{Q} \to \bar{Q}'$ by $g(\{P\}) = [P]$. So,

$$g(f_l(\{q_0\}, \sigma_1)) = g(\{q_0, q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_0\}, \sigma_1)) = f'_l([\{q_0\}], \sigma_1),$$
$$g(f_l(\{q_1\}, \sigma_1)) = g(\{q_0, q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_1\}, \sigma_1)) = f'_l([\{q_1\}], \sigma_1),$$
$$g(f_l(\{q_0, q_1\}, \sigma_1)) = g(\{q_0, q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_0, q_1\}, \sigma_1)) = f'_l([\{q_1\}], \sigma_1),$$
$$g(f_l(\{q_0\}, \sigma_2)) = g(\{q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_0\}, \sigma_2)) = f'_l([\{q_0\}], \sigma_2),$$
$$g(f_l(\{q_1\}, \sigma_2)) = g(\{q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_1\}, \sigma_2)) = f'_l([\{q_1\}], \sigma_2),$$
$$g(f_l(\{q_0, q_1\}, \sigma_2)) = g(\{q_0, q_1\}) = [\{q_1\}] = f'_l((g \times id_X)(\{q_0, q_1\}, \sigma_2)) = f'_l([\{q_1\}], \sigma_2).$$

Then $g : (\bar{Q}, f_l, \delta_l) \to (\bar{Q}', f'_l, \delta'_l)$ is an I-homomorphism. Let $g_{out} : \bar{Z} \to \bar{Z}$ be an identity map. Clearly, $g_{out} \circ \omega_l = \omega'_l \circ g$. Then $\widetilde{F}_l$ and $\widetilde{F}'_l$ are I-morphic. Hence, by considering Theorem 2 and Corollary 1, $\widetilde{F}_l$ and $\widetilde{F}'_l$ have a same behavior.

5. Conclusion

Taking into account the notions of BL-general fuzzy automaton and bisimulation for BL-general fuzzy automaton, in the present study, we defined the notions of BL-intuitionistic general L-fuzzy automaton and I-bisimulation between two BL-intuitionistic general L-fuzzy automata. For a given BL-intuitionistic general L-fuzzy automaton, we obtained the greatest I-bisimulation and the minimal BL-intuitionistic general L-fuzzy automaton. Moreover in this research, the authors showed that if there is an I-bisimulation between two BL-intuitionistic general L-fuzzy automata, then there is a morphism between them so they have the same behavior. Furthermore, we gave an algorithm, which determined the I-bisimulation between any two BL-intuitionistic general L-fuzzy automata.

In the paper "bisimulation of type 2 for BL-general fuzzy automata", we presented bisimulation of type 2 for BL-general fuzzy automaton where bisimulation type 2 was better than the bisimulation of type 1.

Now, we submit two issues as: If there is an I-bisimulation on BL-intuitionistic general L-fuzzy automaton as better than this I-bisimulation which we have presented in this study, and also how we can deal with the idea of I-bisimulation to (fuzzy) pushdown automata and in (fuzzy) tree automata?

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