FUZZY FORCING SET ON FUZZY GRAPHS: DEFINITION AND ITS APPLICATION IN SOCIAL NETWORKS

L. ALIAHMADIPOUR[1], S. RASHIDI*[2]
[1]DEPARTMENT OF COMPUTER SCIENCE
FACULTY OF MATHEMATICS AND COMPUTER,
SHAHID BAHONAR UNIVERSITY OF KERMAN, KERMAN, IRAN
[2]DEPARTMENT OF APPLIED MATHEMATICS,
FACULTY OF MATHEMATICS AND COMPUTER,
SHAHID BAHONAR UNIVERSITY OF KERMAN, KERMAN, IRAN

E-MAILS: L.ALIAHMADIPOUR@UK.AC.IR , SAEEDEH.RASHIDI@UK.AC.IR

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ABSTRACT. Zero forcing is a dynamic process on a graph for changing the color of every vertex to black in an efficient way. This concept is very useful to model opinion formation problem and independent cascade model in social networks. Many type of networks should be modeled by fuzzy graphs. So, we introduce a definition of fuzzy zero forcing set (*fzfs*) on fuzzy graph. Also, we propose an algorithm to construct *fzfs* and compute the propagation time of *fzfs* on fuzzy graphs. Some examples on special fuzzy graphs illustrate the *fzfs*. Also we utilize the *fzfs* in a social network to model opinion formation problem.

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1. INTRODUCTION

Let a graph $G = (V, E)$ a simple undirected graph (no loops, no multiple edges) with a finite nonempty set of vertices V and edge set E (an edge is a two-element

* CORRESPONDING AUTHOR
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subset of vertices). The graphs theory has been applied in many subjects of propagation issues such as the incidence of disease or spread of opinion in a social network. There exist some concepts in the graph theory for propagation problems. One of them is *zero forcing set*. Zero forcing sets and the zero forcing number were introduced in [9]. This definition is based on the *Color-change rule*. Let G be a graph with each vertex colored either white or black, u be a black vertex of G, and exactly one neighbor v of u be white. Then change the color of v to black. When this rule is applied, we say u forces v, and write $u \to v$.

Definition 1.1. *A zero $forcing$ set (or zfs for brevity) of a graph G is a subset Z of vertices such that if initially the vertices in Z are colored black and remaining vertices are colored white, the entire graph G may be colored black by repeatedly applying the color-change rule. The zero forcing number of $G, Z(G)$, is the minimum size of a zero forcing set. Any zero forcing set of order $Z(G)$ is called a minimum zero forcing set* [9].

For a coloring of G, the *derived coloring* is the result of applying the color-change rule until no more changes are possible. For the black set of vertices B, the derived coloring is denoted by $der(B)$ and it is unique [9].

Example 1.2. *In the following figure, we see that $Z(K_n) = n - 1$ and $Z(P_n) =$* 1 [9].

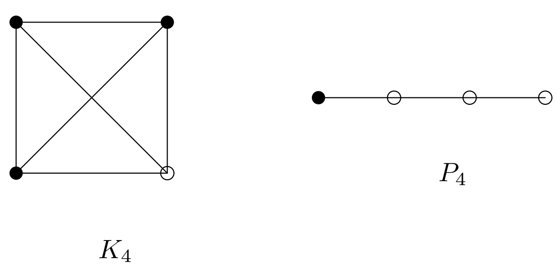

FIGURE 1. $Z(K_4) = 3,\ Z(P_4) = 1.$

Corresponding to the zero forcing sets, there exists one concept as *propagation time* [3].

Definition 1.3. *Let Z be a zero forcing set of G and $Z^0 = Z$. For $t \geq 0$, $Z^{(t+1)}$ is the set of vertices w for which there exists a vertex $b \in \cup_{s=0}^{t} Z^{(s)}$ such that w is the only neighbor of b not in $\cup_{s=0}^{t} Z^{(s)}$. The propagation time of Z in G, denoted $pt(G, Z)$, is the smallest integer t' such that $V = \cup_{s=0}^{t'} Z^{(s)}$* [2].

For example, let Z the black vertices in the Example 1.2. So, $pt(K_4, Z) = 1$ and $pt(P_4, Z) = 3$.

In many examples the graphs are dynamics. All networks can not be modeled by the crisp graphs such as wireless sensor networks (WSN) and social networks. Generally many networks can be modeled by dynamic graphs, random graphs or fuzzy graphs. The social networks can be modeled by the weighted graphs. The best model for them is *fuzzy graph*. The fuzzy graph are introduced in 1975 by Kauffman [5] and are studied by Rosenfeld 1975 [11]. Mordeson gives the following definition of fuzzy graph in [9].

Definition 1.4. *A fuzzy graph is a $G = (V, \sigma, \rho)$ is a nonempty set V together with a pair of functions $\sigma : V \longrightarrow [0, 1]$ and $\rho : V \times V \longrightarrow [0, 1]$ such that for all x, y in V, $\rho(x, y) \leq \sigma(x) \wedge \sigma(y)$. The function σ is the fuzzy vertex set of G and ρ is the fuzzy edge set of G. Note that ρ is a fuzzy relation on σ.*

The *membership function* of a fuzzy set is defined as follows [9]. A fuzzy subset A of a set X is a function $\sigma_A : X \longrightarrow [0, 1]$, this function is also called a membership function. This value, called membership value or degree of membership.
In this paper we propose fuzzy zero forcing set on fuzzy graphs. This new concept is very applicable for social network analysis. For example, in the opinion formation problem the presence, absence or activity of people in the network and changing of the opinions are important issues which can be modeled to propagation in fuzzy graphs. Based on our aim, the rest of paper is organized in the following sections. Section 2 contains the definition of fuzzy zero forcing set and an algorithm to construct this set for fuzzy graphs. In the section 3, we present some examples on special graphs and some remarks. The section 4 investigate the fuzzy zero forcing sets in social networks.

2. Fuzzy zero forcing sets

In this section, we introduce a definition of fuzzy zero forcing set and propose an algorithm to construct this set. We consider applicable-oriented of propagation issue in the definition. Zero forcing sets can be generalized to fuzzy zero forcing sets in three cases as follow:

(1) Considering crisp zero forcing set on fuzzy graph.
(2) Considering fuzzy zero forcing set on crisp graph.
(3) Considering fuzzy zero forcing set on fuzzy graph.

We attend the third case in the general form in the following.

Definition 2.1. *Let $G = (V, \sigma, \rho)$ be a fuzzy graph. Influence rate for each vertex $v_i \in V$ is defined as follows:*

$$I(v_i) = \sigma(v_i)\frac{\sum_{j=1}^{d_i} \rho(v_j, v_i)\sigma(v_j)}{d_i}, \tag{1}$$

where d_i is the degree of v_i in underline graph of G. $\sigma(v_i)$ and $\rho(v_j, v_i)$ are membership value of vertex v_i and edge (v_j, v_i) in G, respectively.

Definition 2.2. *Let $G = (V, \sigma, \rho)$ be a fuzzy graph. Force rate of v on u is computed by the following function:*

$$f(v, u) = \frac{\rho(v, u) - I(u)}{\rho(v, u) + I(u)}. \tag{2}$$

In this case, we measure the amount of force v on u denoted $f(v, u)$.

Definition 2.3. *α is a propagation parameter, where $0 < \alpha \leq 1$.*

We note that α is set based on membership values in G and each application. In the next definition, we introduce a fuzzy zero forcing set constructed using an iterative algorithm.

Definition 2.4. *A fuzzy zero forcing set (fzfs) $\tilde{Z}$, is a fuzzy subset of vertices of G in the following form:*

$$\tilde{Z} = \{\frac{v_i}{\mu_{\tilde{Z}}(v_i)} | \exists j, \mu_{\tilde{Z}}(v_j) = 1\}, \tag{3}$$

where $\tilde{Z}$ is achieved in the following algorithm.

In this definition, we focus on the influence of vertices and connections between them. It means that if the vertices, $\mu_{\tilde{Z}}(v_j) = 1$, are colored black and the remaining vertices are not black, then the black vertices can transform these vertices to black using an iterative algorithm. Note that the proposed algorithm constructs an efficient *fzfs* of a fuzzy graph based on the strength of connection and influence of vertices.

2.1. **Constructing fuzzy zero forcing set. Algorithm 1.**
Input: Fuzzy graph $G = (\sigma, \rho)$ and α.
Output: $\mu_{\tilde{Z}}(v_i) = \mu_{\tilde{Z}^1}(v_i)$.

- **Step 1.** Compute $I(v_i)$ for all $v_i \in V$ and $t = 0$.
- **Step 2. If** $I(v_i) = 1$ for $i = 1, \cdots, n$ then return to crisp mood.
  Else set $\mu_{\tilde{Z}^t}(v_i) = I(v_i)$.
- **Step 3. If** $\exists v_i$ such that $\mu_{\tilde{Z}^t}(v_i) \geq \alpha$ and $\mu_{\tilde{Z}^t}(v_i) \neq 1$
  then $\mu_{\tilde{Z}^{t+1}}(v_i) = 1$ and do:
  - **Step 3.1** Compute $f(v_i, u_j)$ for all white nodes, i.e. $\mu_{\tilde{Z}^t}(u_j) \neq 1$ such that $v_i \in N(u_j)$.
  - **Step 3.2** $\mu_{\tilde{Z}^{t+1}}(u_j) = \max_{v_i}(f(v_i, u_j))$.
  - **Step 3.3** Construct $\tilde{Z}^{t+1} = \{\frac{v_i}{\mu_{\tilde{Z}^{t+1}}(v_i)} | v_i \in V\}$.
  - **Step 3.4 If** $\mu_{\tilde{Z}^{t+1}}(v_i) = 1$ for all $v_i \in V$, then stop and return $\mu_{\tilde{Z}^1}(v_i)$ as *fzfs*.
    Else if $\mu_{\tilde{Z}^{t+1}}(v_i) == \mu_{\tilde{Z}^t}(v_i)$,
    then write ("The fuzzy Graph G doesn't have *fzfs*.")
    Else $t = t + 1$ and go to step 3.

Running Time:
Time complexity of the algorithm depends on the number of iteration, t. The running time of the algorithm is polynomial time of $O(tn^2)$ and we have $t \leq n$.
In the following section, some examples illustrate the new definition (*fzfs*) and the algorithm.

Definition 2.5. *The propagation time of fzfs $\tilde{Z}$ in fuzzy graph G, denoted $\tilde{pt}(G, \tilde{Z})$, is the integer t in the algorithm 1.*

3. Special examples and related results

In this section, we investigate *fzfs* on some graphs such as fuzzy path, fuzzy complete graph and other fuzzy graphs.

Example 3.1. *Let the fuzzy path in the following figure. We set* $\alpha = \min_{i,j} \rho(v_i, v_j)) = 0.2$ *and construct fzfs based on algorithm 1 in the following steps:*

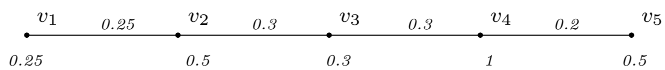

FIGURE 2. A fuuzy path on 5 vertices

(1) ***Step 1.*** $I = \{\frac{v_i}{I(v_i)} | i = 1, \cdots, 5\} = \{\frac{v_1}{0.0312}, \frac{v_2}{0.032}, \frac{v_3}{0.060}, \frac{v_4}{0.095}, \frac{v_5}{0.1}\}$ *and* $t = 0$.

(2) ***Step 2.*** $\tilde{Z}^0 = I$ *and* $\mu_{z^0}(v_i) = I(v_i)$ *for each* v_i.

(3) ***Step 3.*** *For all* $v_i \in V$, $\mu_{z^0}(v_i) \ngeq \alpha$, *So algorithm writes 'The fuzzy Graph G doesn't have fzfs'.*

Example 3.2. *In this example, we let the fuzzy path with some powerful edges. We set* $\alpha = \min_{i,j} \rho(v_i, v_j)) = 0.2$ *and construct fzfs in the following steps:*

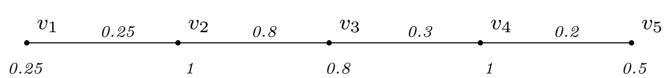

FIGURE 3. A fuuzy path on 5 vertices with powerful edges.

(1) ***Step 1.*** $I = \{\frac{v_1}{0.06}, \frac{v_2}{0.35}, \frac{v_3}{0.88}, \frac{v_4}{0.17}, \frac{v_5}{0.1}\}$ *and* $t = 0$.

(2) ***Step 2.*** $\tilde{Z}^0 = I$ *and* $\mu_{z^0}(v_i) = I(v_i)$ *for each* v_i.

(3) ***Step 3.*** $\tilde{Z}^1 = \{\frac{v_1}{1}, \frac{v_2}{1}, \frac{v_3}{1}, \frac{v_4}{0.1}, \frac{v_5}{0.33}\}$, $t = 1$ *and go Step 3 again.*

(4) ***Step 4.*** $\tilde{Z}^2 = \{\frac{v_1}{1}, \frac{v_2}{1}, \frac{v_3}{1}, \frac{v_4}{1}, \frac{v_5}{1}\}$ *and* $t = 2$. *So* $\tilde{Z} = \tilde{Z}^1$*is a fzfs of this fuzzy graph and* $\tilde{pt}(G, \tilde{Z}) = 2$.

Example 3.3. *Consider the complete fuzzy graph* (K_5) *given in figure 3.3. fzfs of* K_5 *is constructed such as follow: First, we set* $\alpha = \min_{i,j} \rho(v_i, v_j)) = 0.1$.

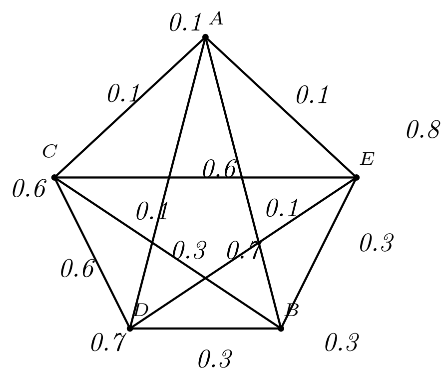

FIGURE 4. A fuzzy complete graph (K_5)

(1) ***Step 1.*** $I = \{\frac{v_i}{I(v_i)} | i = 1, \cdots, 5\} = \{\frac{A}{0.006}, \frac{B}{0.048}, \frac{C}{0.15}, \frac{D}{0.178}, \frac{E}{0.19}\}$ *and* $t = 0$.

(2) ***Step 2.*** $\tilde{Z}^0 = I$ *and* $\mu_{z^0}(v_i) = I(v_i)$ *for each* v_i.

(3) ***Step 3.*** $\tilde{Z}^1 = \{\frac{A}{0.006}, \frac{B}{0.048}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}\}$, $t = 1$ *and go Step 3 again.*

(4) ***Step 4,*** $\tilde{Z}^2 = \{\frac{A}{0.886}, \frac{B}{0.724}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}\}$, *and* $t = 2$ *and go Step 3 again.*

(5) ***Step 5,*** $\tilde{Z}^3 = \{\frac{A}{1}, \frac{B}{1}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}\}$, *and* $t = 3$. *So* $\tilde{Z} = \tilde{Z}^1$*is a fzfs of this fuzzy graph and* $\tilde{pt}(G, \tilde{Z}) = 3$.

Lemma 3.4. *Let* K_n *be a fuzzy complete graph. If* $\sigma(v_1) \leq \sigma(v_2) \cdots \leq \sigma(v_n)$ *then* $I(v_1) \leq I(v_2) \leq \cdots \leq I(v_n)$.

Proof. By definition:

$$I(v_j) = \frac{\sigma(v_j)}{n-1} \sum_{i \neq j} min\{\sigma(v_j), \sigma(v_i)\}\sigma(v_i)$$

$$= \frac{\sigma(v_j)}{n-1}(\sigma(v_1)^2 + \sigma(v_2)^2 + \cdots + \sigma(v_{j-1})^2 + \sum_{i=j+1} \sigma(v_j)\sigma(v_i))$$

and:

$$\begin{aligned}
I(v_{j+1}) &= \frac{\sigma(v_{j+1})}{n-1}(\sigma(v_1)^2 + \sigma(v_2)^2 + \cdots + \sigma(v_j)^2 \\
&+ \sum_{i=j+2} \sigma(v_{j+1})\sigma(v_i)) \\
&= \frac{\sigma(v_{j+1})}{n-1}(\sigma(v_1)^2 + \sigma(v_2)^2 + \cdots + \sigma(v_{j-1})^2) \\
&+ \frac{\sigma(v_j)}{n-1}(\sigma(v_j)\sigma(v_{j+1}) + \frac{\sigma(v_{j+1})}{n-1} \sum_{i=j+2} \sigma(v_{j+1})\sigma(v_i)) \\
&\geq \frac{\sigma(v_j)}{n-1}(\sigma(v_1)^2 + \sigma(v_2)^2 + \cdots + \sigma(v_{j-1})^2) \\
&+ \frac{\sigma(v_j)}{n-1}\sigma(v_j)\sigma(v_{j+1}) + \frac{\sigma(v_j)}{n-1} \sum_{i=j+2} \sigma(v_j)\sigma(v_i) \\
&= \frac{\sigma(v_j)}{n-1}(\sigma(v_1)^2 + \sigma(v_2)^2 + \cdots + \sigma(v_{j-1})^2) \\
&+ \sum_{i=j+1} \sigma(v_j)\sigma(v_i)) = I(v_j)
\end{aligned}$$

□

Example 3.5. *In this example, we let the fuzzy graph illustrated in figure 3.5. fzfs is constructed in the following steps:*

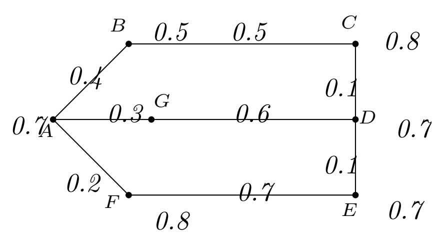

FIGURE 5. A fuzzy graph

(1) ***Step 1.*** $\tilde{Z}^0 = I(V) = \{\frac{A}{0.812}, \frac{B}{0.135}, \frac{C}{0.128}, \frac{D}{0.119}, \frac{E}{0.119}, \frac{F}{0.220}, \frac{G}{0.308}\}$

(2) ***Step 2.*** *For all vertices $I(v_i) > \alpha$, so, propagation time is 1 and we have:*

$$\tilde{Z}^1 = \{\frac{A}{1}, \frac{B}{1}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}, \frac{F}{1}, \frac{G}{1}\}$$

This result illustrates that the fuzzy graph colored to black in two steps because the vertices are strong.

Example 3.6. *In this example, we let the fuzzy star graph with some strong vertices in figure 3.6 . We consider $\alpha = 0.1$ and construct fzfs in the following steps:*

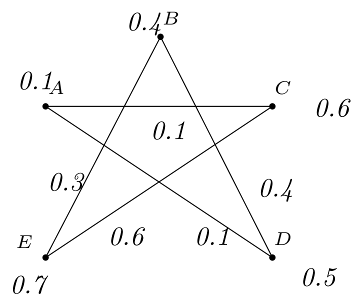

FIGURE 6. A fuzzy star graph

(1) ***Step 1.*** $I = \{\frac{v_i}{I(v_i)} | i = 1, \cdots, 5\} = \{\frac{A}{0.005}, \frac{B}{0.082}, \frac{C}{0.15}, \frac{D}{0.129}, \frac{E}{0.042} \frac{F}{0.168}\}$ *and* $t = 0$.

(2) ***Step 2.*** $\tilde{Z}^0 = I$ *and* $\mu_{z^0}(v_i) = I(v_i)$ *for each* v_i.

(3) ***Step 3.*** $\tilde{Z}^1 = \{\frac{A}{0.005}, \frac{B}{0.082}, \frac{C}{1}, \frac{D}{1}, \frac{E}{0.042}, \frac{F}{1}\}$, $t = 1$ *and go Step 3 again.*

(4) ***Step 4.*** $\tilde{Z}^2 = \{\frac{A}{0.886}, \frac{B}{0.724}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}\}$, *and* $t = 2$ *and go Step 3 again.*

(5) ***Step 5.*** $\tilde{Z}^3 = \{\frac{A}{1}, \frac{B}{1}, \frac{C}{1}, \frac{D}{1}, \frac{E}{1}\}$, *and* $t = 2$. *So* $\tilde{Z} = \tilde{Z}^1$*is a fzfs of this fuzzy graph and* $\tilde{pt}(G, \tilde{Z}) = 3$.

Remark 3.7. *There exists a color-change role in zero forcing set (crisp mode). So, we can propose another definition for fzfs (fuzzy zero forcing set) under the definition of fuzzy coloring. The fuzzy coloring on fuzzy graphs are investigated more. We apply the following definition of fuzzy coloring.*

Definition 3.8. [3] *A family $\Gamma = \gamma_1, \dots, \gamma_k$ of fuzzy sets on X is called a k-fuzzy coloring of $G = (V, \mu, \rho)$ if:*
(a) $\vee\Gamma = \sigma$,
(b) $\gamma_i \wedge \gamma_j = 0$,
(c) for every strong edge xy of G,
$min\ \{\gamma_i, \gamma_j\} = 0,\ (1 \leq i \leq k)$.
The least value of k for which G has a k-fuzzy coloring, denoted by $\chi_f(G)$, is called the fuzzy chromatic number of G.

This Definition can be restricted to crisp graphs, when $\mu = 1$ and $\rho \in \{0,1\}$. By the first condition, the value of color ith for each vertex v is at most $\sigma(v)$. The second condition indicates that each vertex v_i has only one color. By the third condition two vertices of each strong edge can not have the same color. In this definition the aim of strong edge v_iv_j is $\frac{1}{2}(\sigma(v_i) \wedge \sigma(v_j)) \leq \rho(v_i, v_j)$.
We can define a new fuzzy zero forcing set as follows. It is based on the above definition. Here the third condition is not required. Since, in zero forcing process, two adjacent vertices can have the same color. Zero forcing set is dynamical process. Hence, we set the counter t for computing the propagation time.

Definition 3.9. *A family $\Gamma^0 = \{\gamma_0^0, \gamma_1^0\}$ of fuzzy sets on X is called a fuzzy zero forcing set of $G = (V, \mu, \rho)$ if: $t = 0$ and*
(a) $\vee\Gamma^t = \sigma$,
(b) $\gamma_i^t \wedge \gamma_j^t = 0$,
(c) Apply the color change role as crisp mode and $t = t + 1$. The least value of t for which G has a fuzzy zero forcing set is propagation time.

In fact γ_0 is the vertex of fuzzy wight vertex set and γ_1 is the fuzzy black vertex set of fuzzy graph. This type of definition is not more applicable. Since it is very similar to the crisp mood and obtain the zero forcing set by it is exactly similar to the obtain the zero forcing set underline graph for each fuzzy graph.

Example 3.10. *In this example, we check this type of definition for the Example 3.1. This definition does not any attention to the membership of edges. In the first stage we consider the color of v_1 to be black. Now the vertex v_1 is a black vertex of G, and exactly one neighbor of it (the vertex v_2) is white. Then the vertex*

v_1 change the color of the vertex v_2 to black and we continue this process until the color of all vertices of graph change to black

(1) $\gamma_1^0 = \{\frac{v_1}{0.25}, \frac{v_2}{0}, \frac{v_3}{0}, \frac{v_4}{0}, \frac{v_5}{0}\}$
$\gamma_0^0 = \{\frac{v_1}{0}, \frac{v_2}{0.5}, \frac{v_3}{0.3}, \frac{v_4}{1}, \frac{v_5}{0.5}\}$

(2) $\gamma_1^1 = \{\frac{v_1}{0.25}, \frac{v_2}{0.5}, \frac{v_3}{0}, \frac{v_4}{0}, \frac{v_5}{0}\}$
$\gamma_0^1 = \{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0.3}, \frac{v_4}{1}, \frac{v_5}{0.5}\}$

(3) $\gamma_1^2 = \{\frac{v_1}{0.25}, \frac{v_2}{0.5}, \frac{v_3}{0.3}, \frac{v_4}{0}, \frac{v_5}{0}\}$
$\gamma_0^2 = \{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}, \frac{v_4}{1}, \frac{v_5}{0.5}\}$

(4) $\gamma_1^3 = \{\frac{v_1}{0.25}, \frac{v_2}{0.5}, \frac{v_3}{0.3}, \frac{v_4}{1}, \frac{v_5}{0}\}$
$\gamma_0^3 = \{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}, \frac{v_4}{0}, \frac{v_5}{0.5}\}$

(5) $\gamma_1^4 = \{\frac{v_1}{0.25}, \frac{v_2}{0.5}, \frac{v_3}{0.3}, \frac{v_4}{1}, \frac{v_5}{0.5}\}$
$\gamma_0^4 = \{\frac{v_1}{0}, \frac{v_2}{0}, \frac{v_3}{0}, \frac{v_4}{0}, \frac{v_5}{0}\}$

So the fzfs set is $\Gamma^0 = \{\gamma_0^0, \gamma_1^0\}$ and the fuzzy propagation of it is 4.

4. Application of *fzfs* in social networks

A social network is a social structure consist of a set of participants (individuals, organizations,...) and the mutual relations between these participants. Social network analysis is an multidisciplinary endeavour including such fields as social psychology, sociology, statistics, and graph theory [10]. Social networks are naturally modeled as graphs, which we sometimes refer to as a social graph. The entities are the vertices, and an edge connects two vertices if the vertices are related by the relationship that characterizes the network. If there is a degree associated with the relationship, this degree is represented by labeling the edges [6].

The fuzzy graphs are very useful tools to model social networks. In this literature, we have utilized *fzfs* to simulate propagation a desired opinion problem between individuals with the lowest cost in terms of spread time in social networks. Authors in [1] introduced the algorithm to control the opinion formation process by maximization influence of informed agents in social networks. They added a number of informed agents to the network for opinion propagation while we attempt apply the existed entities in the network for opinion propagation. Each people has one opinion [4] in a social network. Jalili introduced the following notation to spread opinion in [4]. The ith agent has opinion $x_i(0)$ at first but it is changing during the time . So, the opinion is a dynamical parameter. Each individual has opinion

$x_i(k)$. This opinion is variable under the influence of other agents. $x_i(k) \in [0,1]$ is the opinion of x_i for one subject in the kth time. So, we can consider $x_i(0)$ as the $\sigma(v_i)$ and the connection between agent i and agent j as $\rho(i,j)$. In this way, the social network can be modeled by a fuzzy graph.

Also, in the social network the opinion at time k, is indicated by $x_i(k)$ can be considered as the value of membership function of fuzzy forcing set for fuzzy graph G at time k. It means that we want the propagation to be at the minimum time. The vertices with the high output degree affect the vertices with same degree and shape opinion of them.

We apply *fzfs* to model opinion formation process in the social network in the following steps:

(1) We consider the black vertices as informed agents in the social network at the first time. Also, the membership function of the vertices of graph is the opinion of each agent, $x_i(0) = \sigma(v_i)$.

(2) The vertices with small input degree is simulated by the vertices that their color is willing to be white. Their color are changed by the black vertices.

(3) Color intensity at time k, $\mu_Z^k(v_i)$, shows the opinion of vertex $x_i(k)$ at kth time. This value can be approximated by $f(u, v_i)$ under the influence of node u.

(4) We run the algorithm 1 on the fuzzy graph G. The output of the algorithm shows that which vertices (agents in the social network) able to propagate the opinion at which time or confirms that the agents based on their connections could not propagate an opinion in the social network.

5. Conclusion and future work

In this paper we introduced fuzzy zero forcing sets that is very useful to model graph coloring and propagation problem such as opinion formation problem and independent cascade model in social networks. We proposed an algorithm to construct *fzfs* and compute the propagation time of *fzfs* on fuzzy graphs. We utilized the *fzfs* in a social network to model opinion formation. The modeling of independent cascade model in details are considered as future work.

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