SOFT α-STRUCTURES IN BI-POLAR SOFT TOPOLOGICAL SPACES

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Abstract. In this current article, soft α-connectedness, soft α-dis-connectedness and soft α-compact spaces in bi-ploar soft topological spaces are discussed with respect to ordinary points. In addition, for better understanding examples have been addressed.

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1. Introduction

A fuzzy set is a class of objects with a continuum of grades of memberships, such a set is characterized by a membership function which assigns to each object a grade of member ship ranging between zero and one [1], A definition of the concept 'intuitionistic fuzzy set' (IFS) is given, the latter being a generalization of the concept 'fuzzy set' and an example is described. Various properties are proved, which are connected to the operations and relations over sets, and with modal and topological operators, defined over the set of IFS's [2], The soft set theory offers a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. The author introduced the basic notions of the theory of soft sets, to present the first results of the theory, and to discuss some problems of the future [3]. Theories which participate in solution of thorny problems such as decision making and uncertainty. Lots of mathematicians tried their hands sophisticatedly on these theories [4, 5, 6, 7].

In the past years, [8] and [9] defined bipolar soft set in variety of ways. Obviously, bipolar soft sets gratify more sharpening results in contrast to soft sets. Therefore the concept of bipolar soft sets exceeds soft sets dramatically. One can confidently say that bipolar soft topology is more effective in practical problems as compared to soft topology.

In this article, we define a short symbolism for writing simplicity in the application of bipolar soft sets and search the linkage between the soft topological structures and the bipolar soft topological structures. soft α-connection, soft α-dis-connection and soft α-compact spaces in bi-ploar soft topological spaces are discussed with respect to ordinary points.The basis theorems of these notations are reflected and supported with suitable examples.

We generate models of realism that are improvement of aspects of the genuine world. These scientific models are excessively convoluted and we cannot locate the analytic solutions. The vulnerability or instability of information while modeling issues in social sciences, medical sciences, artificial intelligence, engineering, natural sciences, etc., makes the utilization of conventional old methods successful. Therefore, traditional set theories, which were based on the crisp and exact case may not be completely suitable for looking after of issues of ambiguity/vagueness. To kick out these instabilities, the soft theory have been proposed [1, 10, 11]. Yet, all these speculations have their natural difficulties. The reason behind these difficulties is, potentially, the insufficiency of the parameterizations instrument of the theory as stated by [12] publicized the notion of the theory of soft sets as a new, effective and powerful mathematical techniques for dealing with instabilities, which is freed from the above challenges. In this article, we exhibited the crucial after effects of the new theory and effectively cemented it to a few headings; for instance game theory, Riemann integration, probability, smoothness of function, and so on. Soft frameworks guide an exceptionally broad system with the contribution of parameters that is attributes or characteristics. Lots of work on soft set theory and its application in different areas have been discussed by a number of researchers [13, 14, 15, 16, 17, 18, 19, 20]. If we review the history of soft topological spaces, the foundation of which was laid by [7], we find many notable authors following them [5, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

The notion of bipolar soft sets (a hybridization of the structure of soft set and bipolarity) with its application in decision making was initiated and discussed in detail by [8] and studied deeply by [9]. A bipolar soft set is acquired by viewing not only a precisely chosen set of parameters, but also an associated set of oppositely meaning parameters called the (not set of parameters)that is, it enjoys the properties of both directions at a time. Due to the quality of providing positive and negative aspects of information at a time, the notion of the bipolar soft set exceeds soft sets. Due to this reality it booms up and got momentum among researchers. [32] applied technically the notion of bipolar soft sets to hemirings. Recently, Shabir and Bakhtawar supposed the study of bipolar soft topological structures. They defined bipolar soft topology as a assembly $\Im$ of bipolar soft sets over the universe of discourse $\sqcup$. Consequently, they defined basic concepts of bipolar soft topological structures, bipolar soft open and bipolar soft closed sets, bipolar soft subspace, bipolar soft closure, bipolar soft interior, bipolar soft neighborhood of a point and addressed their several properties. Further, Shabir and Bakhtawar explored and studied in detail bipolar soft separation axioms.

A. M. Khattak et al. [34, 35] continued work on soft bi-topological structures. The authors discussed weak separation axioms and other separation axioms soft bi-topological space with respect to crisp points and soft points of the spaces respectively.

A. M. Khattak et al. [36, 37, 38] continued has work on another structures known as neutrosophic soft topological structures. The authors introduced generalized and most generalized neutrosophic soft open sets in neutrosophic soft topological structures. They discussed different results with respect to soft point of the space.
In the present situation we inked some newly notion in bipolar soft topological spaces such as bipolar soft α-connected spaces, bipolar soft α-disconnected spaces, bipolar soft α-compact spaces.This article is devoted in to four phases,namely section 1 which is just explained above and section 2, which is devoted to preliminaries on basic concepts related to soft sets, soft topological spaces, bipolar soft sets and bipolar soft topological spaces. Section 3, is devoted to the idea of bipolar soft α-disjoint sets, bipolar soft α-separation of a set, bipolar soft α-connected spaces, bipolar soft α-disconnected spaces and bipolar soft α-hereditary property, and some examples are supposed in favor of these ideas. Section 4, studies the concept of bipolar soft α-compact spaces and some results related to these concepts are exhibited. These newly launched notions in bipolar soft topological spaces will hopefully upgrade the future work and studies to be held in the bipolar soft topology and can be applied beautifully and effectively to push out ambiguities.

2. Preliminary

In this section, we will suppose some preliminary statistics about bipolar soft sets and bipolar soft topological spaces. Let X be an initial universe of discourse and E be a set of parameters(or data or attributes). Let $P(X)$ signifies the power set of X and $\mathbb{A}$, $\mathbb{B}$, $\mathbb{C} \subseteqq E$.

Definition 2.1. [8] *$E = \{\lambda^1, \lambda^2, \lambda^3, ..., \lambda^n\}$ be a set of parameters. The not set of E denoted by $\neg E$ is defined by $\neg E = \{\neg\lambda^1, \neg\lambda^2, \neg\lambda^3, ..., \neg\lambda^n\}$ where for all i, $\neg\lambda^i$ = not λ^i.*

Definition 2.2. [8] *A triplet $(F, G, \mathbb{A})$ is called a bipolar soft set over X, where F and G are mappings, $F : \mathbb{A} \to P(X)$ and $G : \neg\mathbb{A} \to P(X)$ such that $F(\lambda) \sqcap G(\neg\lambda)$ $= \emptyset$ for all $\lambda \in \mathbb{A}$ and $\neg\lambda \in \neg\mathbb{A}$.*

Definition 2.3. [8] *For two bipolar soft sets $(F^1, G^1, \mathbb{A})$ and $(F^2, G^2, \mathbb{B})$ over X, $(F^1, G^1, \mathbb{A})$ is called a bipolar soft subset of $(F^2, G^2, \mathbb{B})$ if*
1. $\mathbb{A} \subseteqq \mathbb{B}$ and

2. $F^1(\lambda) \subseteqq F^2(\lambda)$ *and* $G^2(\neg\lambda) \subseteqq G^1(\neg\lambda)$ *for all* $\lambda \in \mathbb{A}$

This relationship is denoted by $(F^1, G^1, \mathbb{A}) \widetilde{\subseteqq} (F^2, G^2, \mathbb{B})$.

$(F^1, G^1, \mathbb{A})$ *and* $(F^2, G^2, \mathbb{B})$ *are said to be equal if* $(F^1, G^1, \mathbb{A})$ *is a bipolar soft subset of* $(F^2, G^2, \mathbb{B})$ *and* $(F^2, G^2, \mathbb{A})$ *is a bipolar soft subset of* $(F^1, G^1, \mathbb{B})$.

Definition 2.4. [8] *Bipolar soft complement of a bipolar soft set* $(F, G, \mathbb{A})$ *over* X *is denoted by* $(F, G, \mathbb{A})^c$ *and is defined by* $(F, G, \mathbb{A})^c = (F^c, G^c, \mathbb{A})$ *where* $F^c : \mathbb{A} \to P(X)$ *and* $G^c : \neg\mathbb{A} \to P(X)$ *are given by* $F^c(\neg\lambda) = G(\neg\lambda)$ *and* $G^c(\neg\lambda) = F(\lambda)$ *for all* $\lambda \in \mathbb{A}$ *and* $\neg\lambda \in \neg\mathbb{A}$.

Definition 2.5. [8] *Bipolar soft union of two bipolar soft sets* $(F^1, G^1, \mathbb{A})$ *and* $(F^2, G^2, \mathbb{B})$ *over* X *is the bipolar soft set* (H, I, C) *over* X *where* $C = \mathbb{A} \sqcup \mathbb{B}$ *and for all* $\lambda \in C$,

$$H(\lambda) = \begin{cases} F^1(\lambda)\ if\ \lambda \in \mathbb{A} - \mathbb{B} \\ F^2(\lambda)\ if\ \lambda \in \mathbb{B} - \mathbb{A} \\ F^1(\lambda) \sqcup F^2(\lambda)\ if \lambda \in \mathbb{A} \sqcap \mathbb{B}. \end{cases} \tag{1}$$

$$I(\neg\lambda) = \begin{cases} G^1(\neg\lambda)\ if \neg\lambda \in (\neg\mathbb{A}) - (\neg\mathbb{B}) \\ G^2(\neg\lambda)\ if \neg\lambda \in (\neg\mathbb{B}) - (\neg\mathbb{A}) \\ G^1(\neg\lambda) \sqcap G^2(\neg\lambda)\ if \neg\lambda \in (\neg\mathbb{A}) \sqcap (\neg\mathbb{B}). \end{cases} \tag{2}$$

It is denoted by $(F^1, G^1, \mathbb{A}) \widetilde{\sqcup} (F^2, G^2, \mathbb{B}) = (H, I, C)$.

Definition 2.6. [8] *Bipolar soft intersection of two bipolar soft sets* $(F^1, G^1, \mathbb{A})$ *and* $(F^2, G^2, \mathbb{B})$ *over* X *is the bipolar soft set* (H, I, C) *over* X *where*
$C = \mathbb{A} \sqcup \mathbb{B}$ *is non-empty and for all* $\lambda \in C$,
$H(\lambda) = F^1(\lambda) \sqcap F^2(\lambda)$
and
$I(\neg) = G^1(\neg\lambda) \sqcup G^2(\neg\lambda)$.
It is denoted by $(F^1, G^1, \mathbb{A}) \widetilde{\sqcap} (F^2, G^2, \mathbb{B}) = (H, I, C)$.

Definition 2.7. [8] *Let* $(F^1, G^1, \mathbb{A})$ *and* $(F^2, G^2, \mathbb{B})$ *be two bipolar soft sets over* X. *Then*

1. $((F^1, G^1, \mathbb{A}) \widetilde{\sqcup} (F^2, G^2, \mathbb{B}))^c = (F^1, G^1, \mathbb{A})^c \widetilde{\sqcap} (F^2, G^2, \mathbb{B})^c$,

2. $((F^1, G^1, \mathbb{A}) \widetilde{\sqcap} (F^2, G^2, \mathbb{B}))^c = (F^1, G^1, \mathbb{A})^c \widetilde{\sqcup} (F^2, G^2, \mathbb{B})^c$.

Definition 2.8. [8] *A bipolar soft set* $(\digamma, G, \mathbb{A})$ *over* X *is said to be relative null bipolar soft set, denoted by* $(\Phi, \widetilde{X}, \mathbb{A})$*, if for all* $\lambda \in \mathbb{A}$*,* $\digamma(\lambda) = \emptyset$ *and foe all* $\neg\lambda \in \neg\mathbb{A}$*,* $G(\neg\lambda) = X$.
The relative null bipolar soft set with respect to the universe set of parameters E *is called a NULL bipolar soft set over* X *and is denoted by* $(\Phi, \widetilde{X}, E)$.

Definition 2.9. [8] *A bipolar soft set* $(\digamma, G, \mathbb{A})$ *over* X *is said to be relative absolute bipolar soft set, denoted by* $(\widetilde{X}, \Phi, \mathbb{A})$*, if for all* $\lambda \in \mathbb{A}$*,* $\digamma(\lambda) = X$ *and for all* $\neg\lambda \in \neg\mathbb{A}$*,* $G(\neg\lambda) = \emptyset$.
The relative absolute bipolar soft set with respect to the universe set of parameters E *is called a ABSOLUTE bipolar soft set over* X *and denoted by* $(\widetilde{X}, \Phi, E)$.

Definition 2.10. [33] *Let* $\widetilde{\widetilde{\Im}}$ *be the collection of bipolar soft sets over* X *with* E *as the parameters. Then* $\widetilde{\widetilde{\Im}}$ *is said to be a bipolar soft topology over* X *if*
1. $(\Phi, \widetilde{X}, E)$ *and* $(\Phi, \widetilde{X}, E)$ *belong to* $\widetilde{\widetilde{\Im}}$
2. the bipolar soft union of any number of bipolar soft sets in $\widetilde{\widetilde{\Im}}$ *belongs to* $\widetilde{\widetilde{\Im}}$
3. the bipolar soft intersection of finite number of bipolar soft sets in $\widetilde{\widetilde{\Im}}$ *belongs to* $\widetilde{\widetilde{\Im}}$*. Then* $(X, \widetilde{\widetilde{\Im}}, E, \neg E)$ *is called a bipolar soft topological space over* X.

Definition 2.11. [33] *Let* $(X, \widetilde{\widetilde{\Im}}, E, \neg E)$ *be a bipolar soft soft topological space over* X*,then the member of* $\widetilde{\widetilde{\Im}}$ *are said to be bipolar soft open sets in* X.

Definition 2.12. [33] *Let* $(X, \widetilde{\widetilde{\Im}}, E, \neg E)$ *be a bipolar soft topological space over* X*. A bipolar soft set* $(\digamma, G, E)$ *over* X *is said to be a bipolar soft closed set in X, if its bipolar soft complement* $(\digamma, G, E)^c$ *belong to* $\widetilde{\widetilde{\Im}}$.

Definition 2.13. [33] *Let* $(X, \widetilde{\widetilde{\Im}}, E, \neg E)$ *be a bipolar soft topological space over* X*. A bipolar soft set* $(\digamma, G, E)$ *over* X *is said to be a bipolar soft clopen set in X, if it is both a bipolar soft closed set and a bipolar soft open set over* X.

3. Bipolar Soft Weak-(Connected and Disconnected Structures)

In this section is confined to discussion over the most important characteristics of bipolar soft topological spaces called the bipolar soft α-connectedness and bipolar soft α-disconnectedness.

Definition 3.1. *Two bipolar soft sets* $(\digamma^1, G^1, E)$*,* $(\digamma^2, G^2, E)$ *are said to be bipolar soft disjoint if* $\digamma^1(\lambda) \sqcap \digamma^2(\lambda) = \varphi$ *for all* $\lambda \in E$.

Definition 3.2. *Let $(U, \Im, E, \neg E)$ be a bipolar soft topological space over U. A bipolar soft α-separation of (υ, Θ, E) is a pair (F^1, G^1, E), (F^2, G^2, E) of non-null disjoint bipolar soft α-open sets over U such that $F^1(\lambda) \sqcap F^2(\lambda) = U$ for all $\lambda \in E$.*

Definition 3.3. *A bipolar soft topological space $(U, \Im, E, \neg E)$ is said to be a bipolar soft α-disconnected space if there exists a bipolar soft α-separation of (υ, Θ, E). Further, $(U, \Im, E, \neg E)$ is said to be a bipolar soft α-connected space if and only if it is not a bipolar soft α-disconnected space.*

Example 3.4. *Let $U = \{m^1, m^2, m^3, m^4\}$ be the universe set representing "markets".Let $E = \{\lambda^1, \lambda^2\} = \{hand\ embroidery\ dresses,\ formal\ dresses\}$ and $\neg E = \{\neg\lambda^1, \neg\lambda^2\} = \{machine\ embroidery,\ causal\ dresses\}$. Let (F^1, G^1, E), (F^2, G^2, E) represents the preferences of markets for selection of clothes by two women. Then the bipolar soft topology over U generated by (F^1, G^1, E), and (F^2, G^2, E) is given by $\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E)\}$ where (F^1, G^1, E), (F^2, G^2, E),$(F^3, G^3, E)\}$ are bipolar soft sets over U defined as follows:*
$F^1(\lambda^1) = \{m^1\}$, $F^1(\lambda^2) = \{m^2, m^4\}$ and $G^1(\neg\lambda^1) = \{m^2\}$, $G^1(\neg\lambda^2) = \{m^1\}$, $F^2(\lambda^1) = \{m^2, m^3, m^4\}$, $F^2(\lambda^2) = \{m^1, m^3\}$ and $G^2(\neg\lambda^1) = \varphi$, $G^2(\neg\lambda^2) = \{m^2\}$, $F^3(\lambda^1) = \varphi$, $F^3(\lambda^2) = \varphi$, and $G^3(\neg\lambda) = \{m^2\}$, $G^3(\neg\lambda) = \{m^1, m^2\}$. Then $(U, \Im, E, \neg E)$ is a bipolar soft α-disconnected space because (F^1, G^1, E), and (F^2, G^2, E) form a bipolar soft α-separation of (υ, Θ, E).

Example 3.5. *Let $U = \{w^1, w^2, w^3\}$ be the universe set representing"wedding marques". Let $E = \{\lambda^1, \lambda^2, \lambda^3\} = \{Expensive,\ best\ food\ service,\ ideal\ decoration\ facility\}$ be the set of parameters and*
$\neg E = \{\neg\lambda^1, \neg\lambda^2, \neg\lambda^3\} = \{cheap,\ average\ food\ service,\ poor\ decoration\ facility\}$ be the not set of parameters. Let (F^1, G^1, E), (F^2, G^2, E) represents the choices made by two different families for the selection of wedding marques.
Then $\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E)\}$ is the bipolar soft topology over U generated by (F^1, G^1, E), (F^2, G^2, E) where

$$F^1(\lambda^1) = \{w^1, w^3)\},\ F^1(\lambda^2) = \{w^2, w^3)\},\ F^1(\lambda^3) = \{w^1, w^2)\}$$

and

$G^1(\neg\lambda^1) = \{w^2\}$, $G^1(\neg\lambda^2) = \{w^3, w^4\}$, $G^1(\neg\lambda^3) = \{w^3\}$,

$$F^2(\lambda^1) = \{w^3, w^4\},\ F^2(\lambda^2) = \{w^1, w^2, w^3\},\ F^2(\lambda^3) = \{w^1, w^4\}$$

and

$G^2(\neg\lambda^1) = \{w^1, w^2)\}$, $G^2(\neg\lambda^2) = \{w^4\}$, $G^2(\neg\lambda^3) = \varphi$

$$F^3(\lambda^1) = \{w^3\},\ F^3(\lambda^2) = \{w^2, w^3\},\ F^3(\lambda^3) = \{w^1\}$$

and

$G^3(\neg\lambda^1) = \{w^1, w^2\}$, $G^3(\neg\lambda^2) = \{w^3, w^4\}$, $G^3(\neg\lambda^3) = \{w^3\}$,

$$F^4(\lambda^1) = \{w^1, w^3, w^4\},\ F^4(\lambda^2) = \{w^1, w^2, w^3\},\ F^4(\lambda^3) = \{w^1, w^2, w^4\}$$

and

$G^4(\neg\lambda^1) = \{w^2\}$, $G^4(\neg\lambda^2) = \{w^4\}$, $G^4(\neg\lambda^3) = \phi$.

We note that the bipolar soft topological space generated by (F^1, G^1, E), (F^2, G^2, E) *is a bipolar soft* α*-connected space because there does not exist a bipolar soft* α*-separation of* (υ, Θ, E).

Proposition 3.6. *Let* (F, G, E) *be a bipolar soft set. Then*

1) $(F, G, E) \sqcup (F^c, G^c, E) = (H, I, E)$, *where* $H(\lambda) = F(\lambda) \sqcup F^c(\lambda) \subseteqq U$ *for each* $\lambda \in E$ *and*

$I(\neg\lambda) = G(\neg\lambda) \sqcap G^c(\neg\lambda) = \varphi$ *for each* $\neg\lambda \in \neg E$.

2) $(F, G, E) \sqcap (F^c, G^c, E) = (H, I, E)$, *where* $H(\lambda) = F(\lambda) \sqcap F^c(\lambda) = \varphi$ *for each* $\lambda \in E$ *and*

$I(\neg\lambda) = G(\neg\lambda) \sqcup G^c(\neg\lambda) \subseteqq U$ *for each* $\neg\lambda \in \neg E$.

Further (F, G, E), (F^c, G^c, E) *will always satisfy* $F(\lambda) \sqcup F^c(\lambda) = G(\neg\lambda) \sqcup G^c(\neg\lambda)$ *for all* $\lambda \in E$.

3) $(F, G, E) \tilde{\sqcap} (\upsilon, \Theta, E) = (F, G, E)$ *and* $(F, G, E) \tilde{\sqcap} (\upsilon, \Theta, E) = (\upsilon, \Theta, E)$.

Proof. Straight forward. □

Theorem 3.7. *A bipolar soft topological space* $(U, \Im, E, \neg E)$ *is bipolar soft* α*-disconnected space if and only if there exist two bipolar soft* α*-closed sets* (F^1, G^1, E), (F^2, G^2, E), *with* $G^1(\neg\lambda) \neq \varphi$, $G^2(\neg\lambda) \neq \varphi$ *for some* $\lambda \in E$, *such that* $G^1(\neg\lambda) \sqcup G^2(\neg\lambda) = U$ *for all* $\neg\lambda \in \neg E$ *and* $G^1(\neg\lambda) \sqcap G^2(\neg\lambda) = \varphi$ *for all* $\neg\lambda \in \neg E$.

Proof. First, suppose $(U, \Im, E, \neg E)$ is a bipolar soft α-disconnected space. Then there exist a bipolar soft α-separation of (υ, Θ, E). Let (F, G, E) and (H, I, E) forms a bipolar soft α-separation of (υ, Θ, E). Then

$$F(\lambda) \sqcup H(\lambda) = U \; for \; all \; \lambda \in E \tag{3}$$

$$F(\lambda) \neq \varphi \; for \; some \; \lambda \in E \tag{4}$$

$$H(\lambda) \neq \varphi \; for \; some \; \lambda \in E \tag{5}$$

$$F(\lambda) \sqcap H(\lambda) = \varphi \; for \; all \; \lambda \in E \tag{6}$$

As $F(\lambda) = G^c(\neg\lambda)$ and $H(\lambda) = I^c(\neg\lambda)$, therefore from equation (3) we have $G^c(\neg\lambda) \sqcup I^c(\lambda) =$ U. From equation (3), (4) and (5) we have $G^c(\neg\lambda) \sqcap I^c(\lambda) = \varphi$ for all $\neg\lambda \in \neg\lambda$, where $G(\neg\lambda) \neq \varphi$ $I(\neg\lambda) \neq \varphi$ for some $\neg\lambda \in \neg E$.

Further $(F, G, E)^c$ and $(H, I, E)^c$ are bipolar soft α-closed sets, since (F, G, E) and $(H, I, E) \in \Im$.

Conversely, suppose that there exist two bipolar soft α-closed sets (F^1, G^1, E), (F^2, G^2, E) with $G^1(\neg\lambda) \neq \varphi$, $G^2(\neg\lambda) \neq \varphi$ for some $\lambda \in$ E, such that $G^1(\neg\lambda)$ $\sqcup$ $G^2(\neg\lambda) =$ U for all $\neg\lambda \in \neg E$ and $G^1(\neg\lambda) \sqcap G^2(\neg\lambda) = \varphi$ for all $\neg\lambda \in \neg E$.

Then $(F^1, G^1, E)^c$ and $(F^2, G^2, E)^c$ are bipolar soft α-open sets with $(F^1)^c(\lambda) = G^1(\neg\lambda) \neq \varphi$ $(F^2)^c(\lambda) = G^2(\neg\lambda) \neq \varphi$ for some $\lambda \in$ E such that $(F^1)^c(\lambda) \sqcup (F^2)^c(\lambda) = G^1(\neg\lambda) \sqcup G^2(\neg\lambda) =$ U for all $\lambda \in$ E, and $(F^1)^c(\lambda) \sqcap (F^2)^c(\lambda) =\varphi$ for all $\lambda \in$ E. Therefore $(F^1, G^1, E)^c$, and $(F^2, G^2, E)^c$ form a bipolar softα- separation of (υ, Θ, E). Thus $(U, \Im, E, \neg E)$ is a bipolar soft α-disconnected space. □

Remark 3.8. *The union of two bipolar soft α-connected spaces over the same universe need not to be a bipolar soft α-connected space.*

Example 3.9. *Let $U = \{h^1, h^2\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$,*
$\Im^1 = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E)\}$ and $\Im^1 = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^2, G^2, E)\}$ where $F^1(\lambda^1) = U$, $F^1(\lambda^2) = \varphi$ and $G^1(\neg\lambda^1) = \varphi$, $G^1(\neg\lambda^2) = U$, $F^2(\lambda^1) = \varphi$, $F^2(\lambda^2) = U$ and $G^2(\neg\lambda^1) = U$, $G^2(\neg\lambda^2) = \varphi$.
Then $(U, \Im^1, E, \neg E)$, $(U, \Im^2, E, \neg E)$ are bipolar soft α-connected spaces over U. But we note that is not a bipolar soft α-connected space because (F^1, G^1, E) , (F^2, G^2, E) form a bipolar soft α-separation of $(\Phi, \widetilde{u}, E)$ in $\Im^1 \sqcup \Im^2$.

Proposition 3.10. *The intersection of two bipolar soft α-connected spaces over a same universe is a bipolar soft α-connected space.*

Proof. Let $(U, \Im^1, E, \neg E)$ and $(U, \Im^2, E, \neg E)$ be two bipolar soft α-connected spaces. Suppose to the contrary that $(U, \Im^1 \sqcap \Im^2, E, \neg E)$ is not a bipolar soft α-connected space. Then there exist bipolar soft sets (F^1, G^1, E) , (F^2, G^2, E) belonging to $\Im^1 \sqcup \Im^2$, which forms a bipolar soft α-separation of $(\Phi, \widetilde{u}, E)$ in $(U, \Im^1 \sqcap \Im^2, E, \neg E)$. Since (F^1, G^1, E) , $(F^2, G^2, E) \in \Im^1 \sqcap \Im^2$ then (F^1, G^1, E) , $(F^2, G^2, E) \in \Im^1$ and (F^1, G^1, E) , $(F^2, G^2, E) \in \Im^1$. This implies that (F^1, G^1, E) , (F^2, G^2, E) form a bipolar soft α-separation of $(\Phi, \widetilde{u}, E)$ in $(U, \Im^1, E, \neg E)$ and (F^1, G^1, E) , (F^2, G^2, E) form a bipolar soft α-separation of $(\Phi, \widetilde{u}, E)$ in $(U, \Im^2, E, \neg E)$ which is a contradiction to given hypothesis. Thus $(U, \Im^1 \sqcap \Im^2, E, \neg E)$ is a bipolar soft α-connected space. □

Remark 3.11. *The intersection of two bipolar soft α-disconnected spaces over the same universe need not to be a bipolar soft α-disconnected space.*

Example 3.12. *Let* $U = \{h^1, h^2, h^3\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$,
$\Im^1 = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E)\}$ *and*
$\Im^2 = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (H^1, I^1, E), (H^2, I^2, E), (H^3, I^3, E)\}$ *where* $F^1(\lambda^1) = \{h^1\}$,
$F^1(\lambda^2) = \{h^1, h^2\}$, *and* $G^1(\neg\lambda^1) = \{h^2\}$ $G^1(\neg\lambda^2) = \{h^3\}$
$F^2(\lambda^1) = \{h^2, h^3\}$, $F^2(\lambda^2) = \{h^3\}$, *and* $G^2(\neg\lambda^1) = \varphi$, $G^2(\neg\lambda^2) = \{h^1\}$
$F^3(\lambda^1) = \varphi$, $F^3(\lambda^2) = \varphi$ *and* $G^3(\neg\lambda^1) = \{h^2\}$, $G^3(\neg\lambda^2) = \{h^1, h^3\}$
$H^1(\lambda^1) = \{h^1, h^3\}$, $H^1(\lambda^2) = \{h^1, h^3\}$ *and* $I^1(\neg\lambda^1) = \{h^2\}$, $I^1(\neg\lambda^2) = \{h^2\}$
$H^2(\lambda^1) = \{h^2\}$, $H^2(\lambda^2) = \{h^2\}$ *and* $I^2(\neg\lambda^1) = \{h^1\}$, $I^2(\neg\lambda^2) = \{h^1\}$
$H^3(\lambda^1) = \varphi$, $H^3(\lambda^2) = \varphi$ *and* $I^3(\neg\lambda^1) = \{h^1, h^2\}$, $I^3(\neg\lambda^2) = \{h^1, h^2\}$.
Then $(U, \Im^1, E, \neg E)$, $(U, \Im^2, E, \neg E)$ *are bipolar soft α-disconnected spaces over* U. *Now* $\Im^1 \sqcap \Im^2 = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E)\}$. *We note* $(U, \Im^1 \sqcap \Im^2, E, \neg E)$ *is not a bipolar soft α-disconnected space because there do no exist any two non null disjoint bipolar soft α-open sets* (F, G, E) *and* (H, G, E) *belonging to* $\Im^1 \sqcap \Im^2$ *such that* $F(\lambda) \sqcup H(\lambda) = U$ *for all* $\lambda \in E$.

Proposition 3.13. *The union of two bipolar soft α-disconnected spaces over the same universe is a bipolar soft α-disconnected space.*

Proof. Straight forward. □

Theorem 3.14. *Let $(U, \Im, E, \neg E)$ be a bipolar soft topological space over U and let the bipolar soft sets (F^1, G^1, E) and (F^2, G^2, E) form a bipolar soft α-separation of (υ, Θ, E). If $(Y, \Im^Y, E, \neg E)$ is a bipolar soft α-connected subspace of $(U, \Im, E, \neg E)$ then $Y \subseteqq F^1(\lambda)$ for all $\lambda \in E$ or $Y \subseteqq F^2(\lambda)$ for all $\lambda \in E$.*

Proof. Since $(F^1, G^1, E)$and (F^2, G^2, E) form a bipolar soft α-separation of (υ, Θ, E), we have

$$U \sqcap (F^1(\lambda) \sqcup F^2(\lambda)) = U \ for\ each\ \lambda \in E \tag{7}$$

$$F^1(\lambda) \neq \varphi \ for\ some\ \lambda \in E \tag{8}$$

$$F^2(\lambda) \neq \varphi \ for\ some\ \lambda \in E \tag{9}$$

$$F^1(\lambda) \sqcap F^2(\lambda) = \varphi \ for\ all\ \lambda \in E \tag{10}$$

As Y $\subseteqq$ U, we have $({}^Y F^1, {}^Y G^1, E)$ and $({}^Y F^2, {}^Y G^2, E)$ are α-open in $(U, \Im^Y, E, \neg E)$.
From equation (7)
Y $\sqcap$ $(F^1(\lambda) \sqcup F^2(\lambda))$ = Y for each $\lambda \in$ E.
This implies

$$(Y \sqcap (F^1(\lambda)) \sqcup (Y \sqcap F^2(\lambda)) = Y \ for\ all\ \lambda \in E. \tag{11}$$

Also from equation (10)
$(Y \sqcap (F^1(\lambda)) \sqcap (Y \sqcap F^2(\lambda)) = \varphi$ for all $\lambda \in$ E.
As $(Y, \Im^Y, E, \neg E)$ is soft α-connected, so either Y $\sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E or Y $\sqcap F^2(\lambda) = \varphi$ for all λ.
If Y $\sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E then from equation (11) Y $\sqcap F^2(\lambda)$ = Y for all $\lambda \in$ E and this implies Y $\subseteqq F^2(\lambda)$ for all $\lambda \in$ E.
If Y $\sqcap F^2(\lambda) = \varphi$ for all $\lambda \in$ E then from equation (11) Y $\sqcap F^1(\lambda)$ = Y for all $\lambda \in$ E and this implies Y $\subseteqq F^2(\lambda)$ for all $\lambda \in$ E. □

Remark 3.15. *The converse of Theorem 3 does not hold in general.*

Example 3.16. *Let $U = \{h^1, h^2, h^3, h^4\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, and*
$\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E), (F^5, G^5, E), (F^6, G^6, E)\}$, *where*
$F^1(\lambda^1) = \{h^1\}$, $F^1(\lambda^2) = \{h^1\}$ *and* $G^1(\neg\lambda^1) = \{h^2, h^3, h^4\}$, $G^1(\neg\lambda^2) = \{h^2, h^3, h^4\}$,

$F^2(\lambda^1) = \{h^2\}$, $F^2(\lambda^2) = \{h^2\}$ *and* $G^2(\neg\lambda^1) = \{h^1, h^3, h^4\}$, $G^2(\neg\lambda^2) = \{h^1, h^3, h^4\}$,
$F^3(\lambda^1) = \{h^3, h^4\}$, $F^3(\lambda^2) = \{h^3, h^4\}$ *and* $G^3(\neg\lambda^1) = \{h^1, h^2\}$, $G^2(\neg\lambda^2) = \{h^1, h^2\}$,
$F^4(\lambda^1) = \{h^1, h^2\}$, $F^4(\lambda^2) = \{h^1, h^2\}$ *and* $G^4(\neg\lambda^1) = \{h^3, h^4\}$, $G^4(\neg\lambda^2) = \{h^3, h^4\}$,
$F^5(\lambda^1) = \{h^1, h^3, h^4\}$, $F^5(\lambda^2) = \{h^1, h^3, h^4\}$ *and* $G^5(\neg\lambda^1) = \{h^2\}$, $G^5(\neg\lambda^2) = \{h^2\}$,
$F^6(\lambda^1) = \{h^2, h^3, h^4\}$, $F^6(\lambda^2) = \{h^2, h^3, h^4\}$ *and* $G^6(\neg\lambda^1) = \{h^1\}$, $G^6(\neg\lambda^2) = \{h^1\}$.
Then $(Y, \Im, E, \neg E)$ *is a bipolar soft topological space over U. Also, note that* (F^3, G^3, E), (F^4, G^4, E) *from a bipolar soft* α*-separation of* $(\aleph, \Theta, E)$
Now let $Y = \{h^1, h^2\}$ *then* $\{h^1, h^2\}$ *then* $\Im^Y = \{(\Phi, \widetilde{Y}, E), (\widetilde{Y}, \Theta, E), ({}^YF^1, {}^YG^1, E), ({}^YF^2, {}^YG^2, E), ({}^YF^3, {}^YG^3, E), ({}^YF^4, {}^YG^4, E), ({}^YF^5, {}^YG^5, E), ({}^YF^6, {}^YG^6, E), E)\}$ *is a bipolar soft topology over Y, where*
${}^YF^1(\lambda^1) = \{h^1\}$, ${}^YF^1(\lambda^2) = \{h^1\}$ *and* ${}^YG^1(\neg\lambda^1) = \{h^2\}$, ${}^YG^1(\neg\lambda^2) = \{h^2\}$,
${}^YF^2(\lambda^1) = \{h^2\}$, ${}^YF^2(\lambda^2) = \{h^2\}$ *and* ${}^YG^2(\neg\lambda^1) = \{h^1\}$, ${}^YG^2(\neg\lambda^2) = \{h^1\}$,
${}^YF^3(\lambda^1) = \varphi$, ${}^YF^3(\lambda^2) = \varphi$ *and* ${}^YG^3(\neg\lambda^1) = \{h^1, h^2\}$, ${}^YG^3(\neg\lambda^2) = \{h^1, h^2\}$,
${}^YF^4(\lambda^1) = \{h^1, h^2\}$, ${}^YF^4(\lambda^2) = \{h^1, h^2\}$ *and* ${}^YG^4(\neg\lambda^1) = \varphi$, ${}^YG^4(\neg\lambda^2) = \varphi$,
${}^YF^5(\lambda^1) = \{h^1\}$, ${}^YF^5(\lambda^2) = \{h^1\}$ *and* ${}^YG^5(\neg\lambda^1) = \{h^2\}$, ${}^YG^5(\neg\lambda^2) = \{h^2\}$,
${}^YF^6(\lambda^1) = \{h^2\}$, ${}^YF^6(\lambda^2) = \{h^2\}$ *and* ${}^YG^6(\neg\lambda^1) = \{h^1\}$, ${}^YG^6(\neg\lambda^2) = \{h^1\}$,
One can easily note that $Y \subseteqq F^4(\lambda)$ *for all* $\lambda \in E$. *But* $(Y, \Im^Y, E, \neg E)$ *is not a bipolar soft* α*-connected space because* $({}^YF^1, {}^YG^1, E)$ *and* $({}^YF^2, {}^YG^2, E)$ *form a bipolar soft* α*-separation of* $(\widetilde{Y}, \Theta, E)$

Remark 3.17. *If there exist a non null, non-absolute bipolar soft* α*-clopen set over U, then* $(U, \Im, E, \neg E)$ *need not be a bipolar soft* α*-disconnected space.*

Proposition 3.18. *Let* $(U, \Im, E, \neg E)$ *be a bipolar soft topological spaces over U. If there exist a non-null, non-absolute bipolar soft* α*-clopen set* (F, G, E) *over U such that* $F(\lambda) \sqcup F^c(\lambda) = U$ *for each* $\lambda \in E$ *then* $(U, \Im, E, \neg E)$ *is a bipolar soft* α*-disconnected space.*

Proof. Since (F, G, E) is a bipolar soft α-clopen set over U then $(F, G, E)^c$ is a bipolar soft α-clopen set over U. Now, by given hypothesis and by proposition 4, we have $F(\lambda) \sqcup F^c(\lambda) = $ U for all $\lambda \in$ E and $G(\neg\lambda) \sqcap G^c(\neg\lambda) = \varphi$ for each $\neg\lambda \in$ E and
$F(\lambda) \sqcap F^c(\lambda) = \varphi$ for each $\lambda \in$ E and $G(\lambda) \sqcup G^c(\lambda) = $ U for each $\lambda \in$ E.

Therefore, (F, G, E) and $(F, G, E)^c$ form a bipolar soft α-separation of (υ, Θ, E). Thus $(U, \Im, E, \neg E)$ is a bipolar soft α-disconnected space. □

Example 3.19. *Let* $U = \{h^1, h^2, h^3\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, *and* $\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E)\}$ *where* $F^1(\lambda^1) = \{h^1\}$, $F^1(\lambda^2) = \{h^1, h^3\}$ *and* $G^1(\neg\lambda^1) = \{h^2\}$, $G^1(\neg\lambda^2) = \{h^2\}$, $F^2(\lambda^1) = \{h^2\}$, $F^2(\lambda^2) = \{h^2\}$ *and* $G^2(\neg\lambda^1) = \{h^1\}$, $G^2(\neg\lambda^2) = \{h^1, h^3\}$, $F^3(\lambda^1) = \{h^1, h^2\}$, $F^3(\lambda^2) = U$ *and* $G^3(\neg\lambda^1) = \varphi$, $G^3(\neg\lambda^2) = \varphi$, $F^4(\lambda^1) = \varphi$, $F^4(\lambda^2) = \varphi$ *and* $G^4(\neg\lambda^1) = \{h^1, h^2\}$, $G^4(\neg\lambda^2) = U$.
Then $(U, \Im, E, \neg E)$ *is a bipolar soft topological space over U. Note that* (F^1, G^1, E) *is a non-null, non-absolute bipolar soft* α*-clopen set over U but* $(U, \Im, E, \neg E)$ *is not a bipolar soft* α*-disconnected space since there do not exist a bipolar soft* α*-separation of* (υ, Θ, E).

Theorem 3.20. *Let* $(U, \Im^S, E)$ *be a soft topological space over U and* $(U, \Im^B, E, \neg E)$ *be a bipolar soft topological space over U constructed from* $(U, \Im^S, E)$ *as in Theorem db. If* $(U, \Im^S, E)$ *is a soft* α*-disconnected space over U then* $(U, \Im^B, E, \neg E)$ *is a bipolar soft* α*-disconnected space over U.*

Proof. Since $(U, \Im^S, E)$ is a soft α-disconnected space, therefore there exist non soft α-open sets (*say*) (F, E), (H, E) over U such thay
$\widetilde{U}^E = (F, E) \sqcup (H, E)$ and $(F, E) \sqcap (H, E) = \Phi^E$.
Further, (F, G, E), (H, I, E) are non-null bipolar bipolar soft α-open sets (*because* $F(\lambda) \neq \varphi, H(\lambda) \neq \varphi$) where for all $\neg\lambda \in \neg E$, $G(\neg\lambda) = U \backslash F(\lambda)$ and $I(\neg\lambda) = U \backslash H(\lambda)$ since (F, E), (H, E) belongs to $\Im^S$.
Now, $F(\lambda) \sqcup H(\lambda) = U$ for all $\lambda \in E$ and $G(\neg\lambda) \sqcap I(\neg\lambda) = (U \backslash F(\lambda)) \sqcap (U \backslash H(\lambda)) = \varphi$ for each $\lambda \in E$. This implies that (F, G, E), (H, I, E) belonging to $\Im^B$, forms a bipolar soft α-separation of (υ, Θ, E). Thus $(U, \Im^B, E, \neg E)$ is a bipolar soft α-disconnected space. □

Proposition 3.21. *Let* $(Y, \Im', E, \neg E)$ *and* $(Z, \Im'', E, \neg E)$ *be two bipolar soft subspaces of* $(U, \Im, E, \neg E)$ *and let* $Y \subsetneqq$ *.Then* $(Y, \Im', E, \neg E)$ *is a bipolar soft subspace of* $(Z, \Im'', E, \neg E)$.

Proof. As $Y \subsetneqq Z$ so $Y = Y \sqcap Z$. Moreover each bipolar soft α-open set $({}^Y F, {}^Y G, E)$ of $(Y, \Im', E, \neg E)$ is of the form ${}^Y F(\lambda) = Y \sqcap F(\lambda)$ and ${}^Y G(\neg\lambda) = Y \sqcap G(\neg\lambda)$ for

all $\lambda \in$ E where (F, G, E) is a bipolar soft open set of $(U, \Im, E, \neg E)$.
Now for each $\lambda \in$ E,
Y $\sqcap F(\lambda) = (Y \sqcap Z) \sqcap F(\lambda)$ and Y $\sqcap G(\neg\lambda) = (Y \sqcap Z) \sqcap G(\neg\lambda)$.
$\Longrightarrow$ Y $\sqcap F(\lambda) =$ Y $\sqcap (Z \sqcap F(\lambda))$ and Y $\sqcap G(\neg\lambda) =$ Y $\sqcap (Z \sqcap G(\neg\lambda))$.
$\Longrightarrow$ Y $\sqcap F(\lambda) =$ Y $\sqcap {}^Z F(\lambda)$ and Y $\sqcap G(\neg\lambda) =$ Y $\sqcap {}^Z G(\neg\lambda)$ where $({}^Z F, {}^Z G, E)$ is a bipolar soft α-open set in $(Z, \Im'', E, \neg E)$. □

Theorem 3.22. *Let $\{(Y^\alpha, \Im^{Y^\alpha}, E, \neg E)\}^{\alpha \in J}$ be the collection of bipolar soft α-connected subspaces of a bipolar soft topological space $(Y, \Im, E, \neg E)$. If $\sqcap^{\alpha \in J} Y^\alpha \neq \varphi$, then $\sqcup^{\alpha \in J} Y^\alpha, \Im^{\sqcup^{\alpha \in J}} Y^\alpha, E, \neg E\}$ is a bipolar soft α-connected subspace of $(U, \Im, E, \neg E)$.*

Proof. Let $\{(Y^\alpha, \Im^{Y^\alpha}, E, \neg E)\}^{\alpha\, \in\, J}$ be a collection of bipolar soft α-connected subspaces of $(U, \Im, E, \neg E)$, such that $\sqcap^{\alpha \in J} Y^\alpha \neq \varphi$, Suppose that Y $= \sqcup^{\alpha \in J} Y^\alpha$ and $(Y, \Im^Y, E, \neg E)$ be a α-disconnected subspace of $(U, \Im, E, \neg E)$. Let $({}^Y F^1, {}^Y G^1, E)$, $({}^Y F^2, {}^Y G^2, E)$ be a bipolar soft α-separation of (Y, Θ, E). Then

$$ {}^Y F^1(\lambda) \sqcup {}^Y F^2(\lambda) = Y \sqcap (F^1(\lambda) \sqcup (F^2(\lambda)) = Y \; for\ all\ \lambda \in E. \tag{12}$$

$$ Y \sqcap F^1(\lambda) \neq \varphi \; for\ some\ \lambda \in E. \tag{13}$$

$$ Y \sqcap F^2(\lambda) \neq \varphi \; for\ some\ \lambda \in E. \tag{14}$$

$$ (Y \sqcap F^1(\lambda)) \sqcap (Y \sqcap F^2(\lambda)) = Y \sqcap (F^1(\lambda) \sqcap F^2(\lambda)) = \varphi \; for\ all\ \lambda \in E. \tag{15}$$

Consider a fixed Y^α. Then from equation (12)
$Y^\alpha \sqcap (F^1(\lambda) \sqcup F^2(\lambda)) = Y^\alpha$ for all $\lambda \in E$.
From equation (15)
$Y^\alpha \sqcap (F^1(\lambda) \sqcap F^2(\lambda)) = \varphi$ for all $\lambda \in$ E.
Since $(Y^\alpha, \Im^{Y^\alpha}, E, \neg E)$ is a bipolar soft α-connected subspace of $(U, \Im, E, \neg E)$ so either
$Y^\alpha \sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E or $Y^\alpha \sqcap F^2(\lambda) = \varphi$ $\lambda \in$ E.
Now there are three cases:
(i) $Y^\alpha \sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E and for all $\alpha \in$ J
(ii) $Y^\alpha \sqcap F^2(\lambda) = \varphi$ for all $\lambda \in$ E and for all $\alpha \in$ J
(iii) for some $\alpha \in$ J, $Y^\alpha \sqcap F^1(\lambda) = \varphi$ and for other some $\alpha \in$ J, $Y^\alpha \sqcap F^2(\lambda) = \varphi$.
Case: (i)

If $Y^\alpha \sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E and for all $\alpha \in$ J, then $(\sqcup^{\alpha\in J} Y^\alpha) \sqcap F^1(\lambda)) = \varphi$, that is Y $\sqcap F^1(\lambda) = \varphi$ for all $\lambda \in$ E.
This is contradicts equation (13).
Case: *(ii)*
If $Y^\alpha \sqcap F^2(\lambda) = \varphi$ for all $\lambda \in$ E and for all $\alpha \in$ J, then $(\sqcup^{\alpha\in J} Y^\alpha) \sqcap F^2(\lambda)) = \varphi$, that is Y $\sqcap F^2(\lambda) = \varphi$ for all $\lambda \in$ E.
This is contradicts equation (14).
Case: *(iii)*
AS $\sqcap^{\alpha\in J} Y^\alpha \neq \varphi$, so there exist some x $\in Y^\alpha$ for all $\alpha \in$ J. Now by equation (12) x $\in F^1(\lambda) \sqcap F^2(\lambda)$ for all $\lambda \in$ E, this implies x $\in F^1(\lambda)$ or x $\in F^2(\lambda)$.
If x $\in F^1(\lambda)$ then $Y^\alpha \sqcap F^1(\lambda) \neq \varphi$ and if x $\in F^2(\lambda)$ then $Y^\alpha \sqcap F^2(\lambda) \neq \varphi$. So the case *(iii)* is not possible. Hence our supposition is wrong and $(Y, \Im^Y, E, \neg E)$ is a bipolar soft α-connected subspace of $(U, \Im, E, \neg E)$. □

Definition 3.23. *A property P of a bipolar soft topological space $(U, \Im, E, \neg E)$ is said to be a bipolar soft hereditary iff every bipolar soft subspace $(Y, \Im^Y, E, \neg E)$ of $(U, \Im, E, \neg E)$ also possesses the property P.*

Remark 3.24. *The bipolar soft α-connectedness (respect, bipolar soft α-disconnected) is not a bipolar soft hereditary property.*

Example 3.25. *Let $U = \{h^1, h^2, h^3\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, and*
$\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E)\}$
where
$F^1(\lambda^1) = \{h^1\}$, $F^1(\lambda^2) = \{h^1\}$ *and* $G^1(\neg\lambda^1) = \{h^2, h^3\}$, $G^1(\neg\lambda^2) = \{h^2, h^3\}$,
$F^2(\lambda^1) = \{h^2\}$, $F^2(\lambda^2) = \{h^2,\}$ *and* $G^2(\neg\lambda^1) = \{h^1, h^3\}$, $G^2(\neg\lambda^2) = \{h^1, h^3\}$,
$F^3(\lambda^1) = \{h^1, h^2\}$, $F^3(\lambda^2) = \{h^1, h^2\}$ *and* $G^3(\neg\lambda^1) = \{h^3\}$, $G^3(\neg\lambda^2) = \{h^3\}$.
Then $(U, \Im, E, \neg E)$ is a bipolar soft α-connected space.
Now take $Y = \{h^1, h^2\}$. Then $\Im^Y = \{(\Phi, \widetilde{Y}, E), (\widetilde{Y}, \Theta, E), ({}^YF^1, {}^YG^1, E), ({}^YF^2, {}^YG^2, E), ({}^YF^3, {}^YG^3, E)\}$,
where
${}^Y F^1(\lambda^1) = \{h^1\}$, ${}^Y F^1(\lambda^2) = \{h^1\}$ *and* ${}^Y G^1(\neg\lambda^1) = \{h^2\}$, ${}^Y G^1(\neg\lambda^2) = \{h^2\}$,
${}^Y F^2(\lambda^1) = \{h^2\}$, ${}^Y F^2(\lambda^2) = \{h^2\}$ *and* ${}^Y G^2(\neg\lambda^1) = \{h^1\}$, ${}^Y G^2(\neg\lambda^2) = \{h^1\}$,
${}^Y F^3(\lambda^1) = Y$, ${}^Y F^3(\lambda^2) = Y$ *and* ${}^Y G^3(\neg\lambda^1) = \varphi$, ${}^Y G^3(\neg\lambda^2) = \varphi$.

Then $(Y, \Im, E, \neg E)$ is a bipolar soft α-disconnected subspace of bipolar soft α-connected space.

Example 3.26. *Let $U = \{h^1, h^2, h^3\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, and*
$\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E)\}$
where
$F^1(\lambda^1) = \{h^1\}$, $F^1(\lambda^2) = \{h^2\}$ *and* $G^1(\neg\lambda^1) = \{h^2\}$, $G^1(\neg\lambda^2) = \{h^1, h^3\}$,
$F^2(\lambda^1) = \{h^2, h^3\}$, $F^2(\lambda^2) = \{h^1, h^3\}$ *and* $G^2(\neg\lambda^1) = \varphi$, $G^2(\neg\lambda^2) = \{h^2\}$,
$F^3(\lambda^1) = \varphi$, $F^3(\lambda^2) = \varphi$ *and* $G^3(\neg\lambda^1) = \{h^2\}$, $G^3(\neg\lambda^2) = U$.
Then $(U, \Im, E, \neg E)$ is a bipolar soft α-disconnected space.
Now take $Y = \{h^3\}$. Then
$\Im^Y = \{(\Phi, \widetilde{Y}, E), (\widetilde{Y}, \Theta, E), ({}^YF^1, {}^YG^1, E), ({}^YF^2, {}^YG^2, E), ({}^YF^3, {}^YG^3, E)\}$,
where
${}^Y F^1(\lambda^1) = \varphi$, ${}^Y F^1(\lambda^2) = \varphi$ *and* ${}^Y G^1(\neg\lambda^1) = \varphi$, ${}^Y G^1(\neg\lambda^2) = Y$,
${}^Y F^2(\lambda^1) = Y$, ${}^Y F^2(\lambda^2) = Y$ *and* ${}^Y G^2(\neg\lambda^1) = \varphi$, ${}^Y G^2(\neg\lambda^2) = \varphi$,
${}^Y F^3(\lambda^1) = \varphi$, ${}^Y F^3(\lambda^2) = \varphi$ *and* ${}^Y G^3(\neg\lambda^1) = \varphi$, ${}^Y G^3(\neg\lambda^2) = Y$.
Then $(Y, \Im, E, \neg E)$ is a bipolar soft α-connected subspace of bipolar soft α-disconnected space.

4. Bipolar soft Weak-Compact structures

In this section, we study one more eye catching property of bipolar soft topological spaces called the bipolar soft α-compactness. Bipolar soft α-compact spaces are traced and some results related to this notion are derived.

Definition 4.1. *A family $\Psi = \{(F^\alpha, G^\alpha, E)\}^{\alpha\in J}$ of bipolar soft sets is called the bipolar soft cover of a bipolar soft set (F, G, E) if $(F, G, E) \subseteqq \widetilde{\sqcup}^{\alpha\in J}(F^\alpha, G^\alpha, E)$. Further, it is called the bipolar soft open cover of a bipolar soft set (F, G, E) if each member of Ψ is a bipolar soft α-open set. A bipolar soft subcover of Ψ is a subfamily of Ψ which is also a bipolar soft cover.*

Definition 4.2. *A bipolar soft topological space $(U, \Im, E, \neg E)$ is called a bipolar soft α-compact space, if each bipolar soft α-open cover of (υ, Θ, E) has a finite bipolar soft subcover.*

Example 4.3. *Let $U = \{h^1, h^2, h^3\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, and*
$\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E), (F^5, G^5, E),$

$(F^6, G^6, E), (F^7, G^7, E)\}$ *where* (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E), (F^5, G^5, E), (F^6, G^6, E), (F^7, G^7, E) *are bipolar soft sets over U degined as follow:*
$F^1(\lambda^1) = \{h^2\}$, $F^1(\lambda^2) = \{h^2\}$ *and* $G^1(\neg\lambda^1) = \{h^3\}$, $G^1(\neg\lambda^2) = \{h^3\}$,
$F^2(\lambda^1) = \{h^1\}$, $F^2(\lambda^2) = \{h^1\}$ *and* $G^2(\neg\lambda^1) = \{h^3\}$, $G^2(\neg\lambda^2) = \{h^3\}$,
$F^3(\lambda^1) = \{h^1, h^2\}$, $F^3(\lambda^2) = \{h^1, h^2\}$ *and* $G^3(\neg\lambda^1) = \{h^3\}$, $G^3(\neg\lambda^2) = \{h^3\}$,
$F^4(\lambda^1) = \{h^2, h^3\}$, $F^4(\lambda^2) = \{h^2, h^3\}$ *and* $G^4(\neg\lambda^1) = \varphi$, $G^4(\neg\lambda^2) = \varphi$,
$F^5(\lambda^1) = \{h^1, h^3\}$, $F^5(\lambda^2) = \{h^1, h^3\}$ *and* $G^5(\neg\lambda^1) = \varphi$, $G^4(\neg\lambda^2) = \varphi$,
$F^6(\lambda^1) = \varphi$, $F^6(\lambda^2) = \varphi$ *and* $G^6(\neg\lambda^1) = \{h^3\}$, $G^6(\neg\lambda^2) = \{h^3\}$,
$F^7(\lambda^1) = \{h^3\}$, $F^7(\lambda^2) = \{h^3\}$ *and* $G^7(\neg\lambda^1) = \varphi$, $G^7(\neg\lambda^2) = \varphi$.
Then $(U, \Im, E, \neg E)$ *is a bipolar soft topological space over U. Further, we can easily observe that* $(U, \Im, E, \neg E)$ *is a bipolar soft* α*-compact space because every* α*-open cover of* (υ, Θ, E) *has a finite subcover.*

Example 4.4. *Let* $U = N$ *be the universe set of natural numbers, let* $E = \{\lambda^1, \lambda^2\}$ *and* $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$ *be the set of parameters and the not set of parameters, respectively. Let* $\Im$ *be the bipolar soft topology over N,consisting of all bipolar soft sets defined on the parameter set E, generated by the bipolar soft sets* (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E),..., *where*
$F^1(\lambda^1) = \{1, 2\}$, $F^1(\lambda^2) = \{1, 2\}$ *and* $G^1(\neg\lambda^1) = U \setminus \{1, 2\}$, $G^1(\neg\lambda^2) = U \setminus \{1, 2\}$,
$F^2(\lambda^1) = \{2, 3\}$, $F^2(\lambda^2) = \{2, 3\}$ *and* $G^2(\neg\lambda^1) = U \setminus \{2, 3\}$, $G^2(\neg\lambda^2) = U \setminus \{2, 3\}$,
$F^3(\lambda^1) = \{3, 4\}$, $F^3(\lambda^2) = \{3, 4\}$ *and* $G^3(\neg\lambda^1) = U \setminus \{3, 4\}$, $G^3(\neg\lambda^2) = U \setminus \{3, 4\}$,
,...,
$F^n(\lambda^1) = \{n, n+1\}$, $F^n(\lambda^2) = \{n, n+1\}$ *and* $G^n(\neg\lambda^1) = U \setminus \{n, n+1\}$, $G^n(\neg\lambda^2) = U \setminus \{n, n+1\}$
...
Then the bipolar soft topological space $(U, \Im^1, E, \neg E)$ *over U generated by the bipolar soft sets* $\{(F^n, G^n, E) : n \in N\}$ *is not a bipolar soft* α*-compact space since* $\Psi = \{(F^n, G^n, E) : n \in N\}$ *is a bipolar soft* α*-open cover of N with no finite bipolar soft subcover.*

Definition 4.5. *Let* $(U, \Im^1, E, \neg E)$ *and* $(U, \Im^2, E, \neg E)$ *be two bipolar soft topological spaces over the universe U. If* $\Im^1 \subseteqq \Im^2$, *Then* $\Im^2$ *is said to be finer then* $\Im^1$. *If* $\Im^1 \subseteqq \Im^2$ *or* $\Im^2 \subseteqq \Im^1$, *Then* $\Im^1$ *is comparable with* $\Im^2$.

Proposition 4.6. *Let $(U, \Im^2, E, \neg E)$ be a bipolar soft α-compact space and $\Im^1 \subseteqq \Im^2$. Then $(U, \Im^1, E, \neg E)$ is a bipolar soft α-compact.*

Proof. Let $\{(F^\alpha, G^\alpha, E)\}^{\alpha \in J}$ be the bipolar soft α-open cover of (υ, Θ, E) in $(U, \Im^1, E, \neg E)$.Since $\Im^1 \subseteqq \Im^2$, then $\{(F^\alpha, G^\alpha, E)\}^{\alpha \in J}$ is the bipolar soft α-open cover of (υ, Θ, E) by bipolar soft α-open sets of $(U, \Im^2, E, \neg E)$. But $(U, \Im^2, E, \neg E)$ is a bipolar soft α-compact space.
Therefore $(U, \Theta, E) \widetilde{\subseteqq} (F^\alpha 1, G^\alpha 1, E) \widetilde{\sqcup} (F^\alpha 2, G^\alpha 2, E) ... \widetilde{\sqcup} (F^\alpha n, G^\alpha n, E)$, for some α^1,α^2,...,$\alpha^n \in$ J. Hence $(U, \Im^1, E, \neg E)$ is a bipolar soft α-compact space. □

Theorem 4.7. *Let $(Y, \Im^Y, E, \neg E)$ be a bipolar soft subspace of $(U, \Im, E, \neg E)$. Then $(Y, \Im^Y, E, \neg E)$ is a bipolar soft α-compact space if and only if every cover of $(\widetilde{Y}, \Theta, E)$ by bipolar soft α-open sets in U contains a finite subcover.*

Proof. Let $(Y, \Im^Y, E, \neg E)$ be a bipolar soft α-compact space and $\{(F^\alpha, G^\alpha, E)\}^{\alpha \in J}$ be a cover of $(\widetilde{Y}, \Theta, E)$ by bipolar soft α-open sets in U. Now, Y $\subseteqq \sqcup^{\alpha \in J} (Y \sqcap F^\alpha(\lambda))$ for each $\lambda \in$ E, and ? $\subseteqq (Y \sqcap G^\alpha(\neg E))$ for each $\neg\lambda \in \neg E$.
Therefore, $\{({}^Y F^\alpha, {}^Y G^\alpha, E)\}^{\alpha \in J}$ is a bipolar soft α-open cover of (Y, Θ, E).
Since $(Y, \Im^Y, E, \neg E)$ is a bipolar soft α-compact space, therefore, we have
$(\widetilde{Y}, \Theta, E) \widetilde{\subseteqq} \{({}^Y F^{\alpha^1}, {}^Y G^{\alpha^1}, E)\}$
$\widetilde{\sqcup}\{({}^Y F^{\alpha^2}, {}^Y G^{\alpha^2}, E)\}\widetilde{\sqcup}...\widetilde{\sqcup}\{({}^Y F^{\alpha^n}, {}^Y G^{\alpha^n}, E)\}$ for some α^1,α^2,...,$\alpha^n \in$ J.
This implies that $\{(F^{\alpha^i}, G^{\alpha^i}, E)\}_{i=1}^{n}$ is a bipolar subcover of $(\widetilde{Y}, \Theta, E)$ by bipolar soft α-open sets in U.
Conversely, suppose $\{({}^Y F^\alpha, {}^Y G^\alpha, E)\}^{\alpha \in J}$ is a bipolar soft α-open cover of $(\widetilde{Y}, \Theta, E)$.
It is easy to see that $\{(F^\alpha, G^\alpha, E)\}^{\alpha \in J}$ is a bipolar soft α-open cover of $(\widetilde{Y}, \Theta, E)$ by bipolar soft α-open sets in U. Therefore, by given hypothesis we have
$(\widetilde{Y}, \Theta, E)\widetilde{\subseteqq}\{({}^Y F^{\alpha^1}, {}^Y G^{\alpha^1}, E)\}\widetilde{\sqcup}\{({}^Y F^{\alpha^2}, {}^Y G^{\alpha^2}, E)\}\widetilde{\sqcup}...\widetilde{\sqcup}\{({}^Y F^{\alpha^n}, {}^Y G^{\alpha^n}, E)\}$ for some α^1,α^2,...,$\alpha^n \in J$. Thus, $\{(F^\alpha i, G^\alpha i, E)\}_{i=1}^{n}$ is a bipolar subcover of $(\widetilde{Y}, \Theta, E)$.
Hence $(Y, \Im^Y, E, \neg E)$ is a bipolar soft α-compact space. □

Definition 4.8. *Let $(U, \Im^Y, E, \neg E)$ be a bipolar soft topological space over U and let $\beta \subseteqq \Im$. If every element of $\Im$ can be written as the union of the elements of β, then β is called a bipolar soft basis for bipolar soft topology $\Im$. Each element of β is called a bipolar soft basis element.*

Example 4.9. *Let $U = \{h^1, h^2, h^3, h^4\}$, $E = \{\lambda^1, \lambda^2\}$, $\neg E = \{\neg\lambda^1, \neg\lambda^2\}$, and*
$\Im = \{(\Phi, \widetilde{u}, E), (\upsilon, \Theta, E), (F^1, G^1, E), (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E),$

$(F^5, G^5, E), (F^6, G^6, E), (F^7, G^7, E), (F^8, G^8, E)\}$ *where* (F^2, G^2, E), (F^3, G^3, E), (F^4, G^4, E), (F^5, G^5, E), (F^6, G^6, E),(F^7, G^7, E), (F^8, G^8, E) *are bipolar soft sets over U defined as follow:*

$F^1(\lambda^1) = \{h^1, h^2, h^3\}$, $F^1(\lambda^2) = \{h^1, h^2\}$ *and* $G^1(\neg\lambda^1) = \{h^4\}$, $G^1(\neg\lambda^2) = \{h^3, h^4\}$,
$F^2(\lambda^1) = \{h^1, h^2, h^4\}$, $F^2(\lambda^2) = \{h^1, h^2\}$ *and* $G^2(\neg\lambda^1) = \{h^3\}$, $G^2(\neg\lambda^2) = \{h^3, h^4\}$,
$F^3(\lambda^1) = \{h^3, h^4\}$, $F^3(\lambda^2) = \varphi$ *and* $G^3(\neg\lambda^1) = \{h^1, h^2\}$, $G^3(\neg\lambda^2) = U$,
$F^4(\lambda^1) = \{h^1, h^2\}$, $F^4(\lambda^2) = \{h^1, h^2\}$ *and* $G^4(\neg\lambda^1) = \{h^3, h^4\}$, $G^4(\neg\lambda^2) = \{h^3, h^4\}$,
$F^5(\lambda^1) = \{h^3\}$, $F^5(\lambda^2) = \varphi$ *and* $G^5(\neg\lambda^1) = \{h^1, h^2, h^4\}$, $G^5(\neg\lambda^2) = U$,
$F^6(\lambda^1) = \{h^4\}$, $F^6(\lambda^2) = \varphi$ *and* $G^6(\neg\lambda^1) = \{h^1, h^2, h^4\}$, $G^6(\neg\lambda^2) = U$,
$F^7(\lambda^1) = U$, $F^7(\lambda^2) = \{h^1, h^2\}$ *and* $G^7(\neg\lambda^1) = \varphi$, $G^7(\neg\lambda^2) = \{h^3, h^4\}$
$F^8(\lambda^1) = \{h^3, h^4\}$, $F^8(\lambda^2) = \{h^3, h^4\}$ *and* $G^8(\neg\lambda^1) = \{h^1, h^2\}$, $G^8(\neg\lambda^2) = \{h^1, h^2\}$.
Then $(U, \Im, E, \neg E)$ *is a bipolar soft topological space over U. Now if we take* $\beta = \{(\Phi, \widetilde{u}, E), (F^4, G^4, E),$
$(F^5, G^5, E), (F^6, G^6, E), (F^7, G^7, E), (F^8, G^8, E)\}$, *then* β *is a bipolar soft basis for* $\Im$.
Next, if we take $\beta' \subsetneqq \Im$ *where* $\beta' = \{(\Phi, \widetilde{u}, E), (F^1, G^1, E), (F^4, G^4, E), (F^5, G^5, E), (F^6, G^6, E)\}$ *then* β' *is not a bipolar soft* α*-basis because* (υ, Θ, E) *con not be written as the union of the elements of* β'.

Theorem 4.10. *A bipolar soft topological space* $(U, \Im, E, \neg E)$ *is a bipolar soft* α*-compact space if and only if there is a bipolar soft* α*-basis* β *for* $\Im$ *such that every bipolar soft cover of* (υ, Θ, E) *by the elements of* β *has a finite bipolar soft subcover.*

Proof. Let $(U, \Im, E, \neg E)$ be a bipolar soft α-compact space. Obviously $\Im$ is a bipolar soft α-basis for $\Im$. Therefore, every bipolar soft cover of (υ, Θ, E) by elements of $\Im$ has a finite bipolar soft subcover.
Conversely, to show $(U, \Im, E, \neg E)$ is a bipolar soft The credit of strengthening the foundations in the tool box of bipolar soft topology will be given to these newly defined concepts-compact, let $\{(L^\alpha, M^\alpha, E)\}^{\alpha \in J}$ be a bipolar soft α-open cover of (υ, Θ, E). We can write (L^α, M^α, E) as a union of basis element for each $\alpha \in J$. These elements form a bipolar soft α-open cover of (υ, Θ, E) such that $\{(F^\beta, G^\beta, E)\}^{\beta \in J}$. Now, by given hypothesis, for some β^1,β^2,,...,$\beta^n \in$ I, we have
$U = F^{\beta^1}(\lambda) \sqcup F^{\beta^2}(\lambda) \sqcup F^{\beta^3}(\lambda)...\sqcup F^{\beta^n}(\lambda)$, for each $\lambda \in$ E and
$\varphi = G^{\beta^1}(\neg\lambda) \sqcap G^{\beta^2}(\neg\lambda) \sqcap ... \sqcap G^{\beta^n}(\neg\lambda)$ for each $\neg\lambda \in \neg E$. That is $(\upsilon, \Theta, E) = (F^{\beta^1}, G^{\beta^1}, E) \,\widetilde{\sqcup}\, (F^{\beta^2}, G^{\beta^2}, E) \,\widetilde{\sqcup}...\widetilde{\sqcup}\, (F^\beta n, G^\beta n, E)$ for some β^1,β^2,...,$\beta^n \in$ I. Now,

$(F^{\beta^1}, G^{\beta^1}, E) \widetilde{\subseteqq} (L^{\alpha^1}, M^{\alpha^2}, E)$, for each $1 \leq$ i $\leq$ n. This implies that $\{(L^{\alpha^i}, M^{\alpha^i}, E)\}_{i=1}^{n}$ is a finite bipolar subcover of (υ, Θ, E). Hence $(U, \Im, E, \neg E)$ is a bipolar soft α-compact space. □

5. Conclusion

The characteristics of bipolar soft α-connected spaces, bipolar soft α-disconnected spaces and bipolar soft α-compact spaces are addressed with respect to crisp points. The findings and results which we have reflected can be applied to all these problem which contain uncertainty.

6. Completing Interests

The authors declare that there are no competing interests.

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