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(3,2)-FUZZY UP (BCC)-SUBALGEBRAS AND (3.2)-FUZZY UP (BCC)-FILTERS

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Dedicated to sincere professor Mashaallah Mashinchi

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Abstract. The aim of this article is to apply a $(3,2)$-fuzzy set to the UP (BCC)-subalgebras and UP (BCC)-filters of UP (BCC)-algebras. The concepts of $(3,2)$-fuzzy UP (BCC)-subalgebra, $(3,2)$-fuzzy near UP (BCC)-filter and $(3,2)$-fuzzy UP (BCC)-filter in UP (BCC)-algebras are introduced and several properties, including their relations, are investigated. The conditions under which the $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter) can be the $(3,2)$-fuzzy near UP (BCC)-filter (resp., $(3,2)$-fuzzy UP (BCC)-filter) are searched. Some characterizations of $(3,2)$-fuzzy UP (BCC)-filter is provided and the relationship between intuitionistic fuzzy UP-subalgebra and $(3,2)$-fuzzy UP (BCC)-subalgebra is discussed. We use fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP-filter) to create a $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter, $(3,2)$-fuzzy UP (BCC)-filter).

Keywords: $(3,2)$-fuzzy UP (BCC)-subalgebra, $(3,2)$-fuzzy near UP (BCC)-filter, $(3,2)$-fuzzy UP (BCC)-filter, intuitionistic fuzzy UP-subalgebra.
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1. Introduction

Atanasov is a Bulgarian mathematician and a member of the Bulgarian Academy of Sciences. He is best known for introducing the concept of intuitionistic fuzzy sets, which is one of the fuzzy set extensions with better applications such as medical diagnostics, optimization challenges, and multi-criteria decision making, etc. (see [1–4]). Yager [15] offered a new fuzzy set called a Pythagorean fuzzy set, which is a generalization of intuitionistic fuzzy sets. Ibrahim et al. defined the notion of a $(3,2)$-fuzzy set, which is also a generalization of intuitionistic fuzzy sets, and compared it with other kinds of fuzzy sets, and then they discussed topological space based on $(3,2)$-fuzzy sets (see [8]). Iampan introduced the concept of UP-algebras (see [6]) as a generalization of KU-algebras (see [12]) and investigated their properties. Later several substructures of UP-algebras have been discussed by several researchers

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(see [7,9]). Somjanta et al. applied the concept of fuzzy sets to UP-algebras and investigated various properties (see [14]). Kesorn et al. [10] applied the concept of intuitionistic fuzzy sets to UP-algebras and investigated various properties.

In developing a new algebraic structure, everyone has to study the existing algebraic structure seriously and in depth, but the author A. Iampan of UP-algebras seems to have neglected the study of the existing algebraic structure. This is because the concept of UP (BCC)-algebras was first introduced by Komori in 1984 (see [11]), and it can be observed that UP-algebras are the same as UP (BCC)-algebras. In general, we think that the basic virtue of the researcher is to use the concept first introduced when using the existing concept in the study of algebraic structures. So in this paper, we will use the concept of UP (BCC)-algebras instead of the concept of UP-algebras.

In this study, we are going to apply a $(3,2)$-fuzzy set to the UP (BCC)-subalgebras and UP (BCC)-filters of UP (BCC)-algebras. The main points of this study are as follows.

(1) Introducing the concepts of $(3,2)$-fuzzy UP (BCC)-subalgebra, $(3,2)$-fuzzy near UP (BCC)-filter and $(3,2)$-fuzzy UP (BCC)-filter in UP (BCC)-algebras, and investigating related properties.
(2) To discuss relationship between $(3,2)$-fuzzy UP (BCC)-subalgebra, $(3,2)$-fuzzy near UP (BCC)-filter, and $(3,2)$-fuzzy UP (BCC)-filter.
(3) To provide conditions under which the structure becomes $(3,2)$-fuzzy UP (BCC)-subalgebra and $(3,2)$-fuzzy near UP (BCC)-filter.
(4) Providing conditions that $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter) becomes $(3,2)$-fuzzy near UP (BCC)-filter (resp., $(3,2)$-fuzzy UP (BCC)-filter).
(5) Considering the characterization of $(3,2)$-fuzzy UP (BCC)-filter.
(6) Discussting the relationship between intuitionistic fuzzy UP-subalgebra and $(3,2)$-fuzzy UP (BCC)-subalgebra.
(7) Deriving $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter, $(3,2)$-fuzzy UP (BCC)-filter) from a given fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP-filter).

2. **Preliminaries**

This section provides the basics required for this research.

The concept of BCC-algebra was first introduced by Komori in 1984 as follows.

Definition 2.1 ([11]). A *BCC-algebra* is an algebra $A := (A; \to, 1)$ of type (2,0) such that for every $x, y, z \in A$ the following conditions are satisfied:
(BCC-1) $(y \to z) \to ((x \to y) \to (x \to z)) = 1$,
(BCC-2) $x \to x = 1$,
(BCC-3) $x \to 1 = 1$,
(BCC-4) $1 \to x = x$,
(BCC-5) If $x \to y = 1$ and $y \to x = 1$, then $x = y$.

Remark 2.2. The condition (BCC-2) may be caused by the other conditions. In fact, if we put $x = y = 1$ and $z = x$ in (BCI-1), then

$$1 = (1 \to x) \to ((1 \to 1) \to (1 \to x)) = x \to (1 \to x) = x \to x$$

by (BCC-4).

By Remark 2.2, the BCC-algebra can be newly defined without condition (BCC-2).

Definition 2.3 ([5], p.33)**.** Let X be a set with a special element 0 and a binary operation " $*$ ". Then $X := (X, *, 0)$ is called a *BCC-algebra* if it satisfies:

(C1) $(\forall x, y, z \in X)\ ((y * z) * ((x * y) * (x * z)) = 0)$,
(C2) $(\forall x \in X)\ (x * 0 = 0)$,
(C3) $(\forall x \in X)\ (0 * x = x)$,
(C4) $(\forall x, y \in X)\ (x * y = 0,\ y * x = 0 \ \Rightarrow\ x = y)$.

In 2017, Iampan introduced the concept of UP-algebras in [6] as follows.

Definition 2.4 ([6])**.** An algebra $X = (X, *, 0)$ of type $(2, 0)$ is called a *UP-algebra* if it satisfies the following conditions.

(1) $$(\forall x, y, z \in X)((y * z) * ((x * y) * (x * z)) = 0),$$

(2) $$(\forall x \in X)(0 * x = x),$$

(3) $$(\forall x \in X)(x * 0 = 0),$$

(4) $$(\forall x, y \in X)(x * y = 0 = y * x \ \Rightarrow\ x = y).$$

We can observe that the concept of UP-algebras and the concept of BCC-algebras are exactly the same. So in this paper, we would like to honor Komori's achievements by using the the concept of BCC-algebras instead of the concept of UP-algebras.

We define a binary relation " $\leq$ " on a BCC-algebra X as follows:

(5) $$(\forall x, y \in X)\left(\ x \leq y \ \Leftrightarrow\ x * y = 0\ \right).$$

In a BCC-algebra X, the following assertions are valid (see [6]).

(6) $$(\forall x \in X)(x * x = 0),$$

(7) $$(\forall x, y, z \in X)(x * y = 0,\ y * z = 0 \ \Rightarrow\ x * z = 0),$$

(8) $$(\forall x, y, z \in X)(x * y = 0 \ \Rightarrow\ (z * x) * (z * y) = 0),$$

(9) $$(\forall x, y, z \in X)(x * y = 0 \ \Rightarrow\ (y * z) * (x * z) = 0),$$

(10) $$(\forall x, y \in X)(x * (y * x) = 0),$$

(11) $$(\forall x, y \in X)((y * x) * x = 0 \ \Leftrightarrow\ x = y * x),$$

(12) $$(\forall x, y \in X)(x * (y * y) = 0).$$

A subset F of a UP (BCC)-algebra X is called

- a *UP-subalgebra* of X (see [6]) if it satisfies the following condition.

$$(\forall x, y \in X)(x \in F,\ y \in F \ \Rightarrow\ x * y \in F). \tag{13}$$

- a *UP-filter* of X (see [14]) if it satisfies the following conditions.

$$0 \in F, \tag{14}$$

$$(\forall x, y \in X)(x \in F,\ x * y \in F \ \Rightarrow\ y \in F). \tag{15}$$

- a *near UP-filter* of X (see [7]) if it satisfies (14) and

$$(\forall x, y \in X)(y \in F \ \Rightarrow\ x * y \in F). \tag{16}$$

Let X be a UP (BCC)-algebra. A fuzzy set $\mu : X \to [0, 1]$ is called

- a *fuzzy UP-subalgebra* of X (see [14]) if it satisfies:

$$(\forall x, y \in X)(\mu(x * y) \geq \min\{\mu(x), \mu(y)\}), \tag{17}$$

- a *fuzzy UP-filter* of X (see [14]) if it satisfies:

$$(\forall x \in X)(\mu(0) \geq \mu(x)), \tag{18}$$

$$(\forall x, y \in X)(\mu(y) \geq \min\{\mu(x), \mu(x * y)\}), \tag{19}$$

- a *fuzzy near UP-filter* of X (see [13]) if it satisfies (18) and

$$(\forall x, y \in X)(\mu(x * y) \geq \mu(y)). \tag{20}$$

Let X be a nonempty set. Consider a structure

$$\mathcal{E}_X := \{\langle x, f(x), g(x)\rangle \mid x \in X\}, \tag{21}$$

where $f : X \to [0, 1]$ is the degree of membership of x to $\mathcal{E}$ and $g : X \to [0, 1]$ is the degree of non-membership of x to $\mathcal{E}$. The structure in (21) is simply denoted by $\mathcal{E}_X := (X, f, g)$.

A structure $\mathcal{E}_X := (X, f, g)$ is called

- an *intuitionistic fuzzy set* on X (see [1]) if it satisfies:

$$(\forall x \in X)(0 \leq f(x) + g(x) \leq 1), \tag{22}$$

- a $(3, 2)$-*fuzzy set* on X (see [8]) if it satisfies:

$$(\forall x \in X)(0 \leq (f(x))^3 + (g(x))^2 \leq 1). \tag{23}$$

In what follows, we use the notations $f^m(x)$ and $g^n(x)$ instead of $(f(x))^m$ and $(g(x))^n$ for $(m, n) = (3, 2)$, respectively.

A structure $\mathcal{E}_X := (X, f, g)$ on a BCC-algebra X is called an *intuitionistic fuzzy UP-subalgebra* of X (see [10]) if it satisfies:

$$(\forall x, y \in X) \begin{pmatrix} f(x * y) \geq \min\{f(x), f(y)\} \\ g(x * y) \leq \max\{g(x), g(y)\} \end{pmatrix}. \tag{24}$$

3. $(3,2)$-fuzzy UP (BCC)-subalgebras/UP (BCC)-filters

In this section, we define the notions of $(3,2)$-fuzzy UP (BCC)-subalgebras and $(3,2)$-fuzzy (near) UP (BCC)-filters, and explore several properties, relations and characterization etc.

In what follows, let X denote a UP (BCC)-algebra unless otherwise specified.

Definition 3.1. A structure $\mathcal{E}_X := (X, f, g)$ on X is called

- a $(3,2)$-*fuzzy BCC-subalgebra* of X if it satisfies:

$$(\forall x, y \in X)\left(\begin{array}{l} f^3(x*y) \geq \min\{f^3(x), f^3(y)\} \\ g^2(x*y) \leq \max\{g^2(x), g^2(y)\} \end{array}\right). \tag{25}$$

- a $(3,2)$-*fuzzy BCC-filter* of X if it satisfies:

$$(\forall x \in X)(f^3(0) \geq f^3(x),\ g^2(0) \leq g^2(x)), \tag{26}$$

$$(\forall x, y \in X)\left(\begin{array}{l} f^3(y) \geq \min\{f^3(x*y), f^3(x)\} \\ g^2(y) \leq \max\{g^2(x*y), g^2(x)\} \end{array}\right). \tag{27}$$

- a $(3,2)$-*fuzzy near BCC-filter* of X if it satisfies (26) and

$$(\forall x, y \in X)\left(\begin{array}{l} f^3(x*y) \geq f^3(y) \\ g^2(x*y) \leq g^2(y) \end{array}\right). \tag{28}$$

Example 3.2. *(1) Let $X = \{0, a, b, c\}$ be a set with the binary operation "$*$" given in the following table:*

$*$	0	a	b	c
0	0	a	b	c
a	0	0	a	c
b	0	0	0	c
c	0	a	a	0

Then X is a UP (BCC)-algebra. Define a structure $\mathcal{E}_X := (X, f, g)$ on X by the table below:

X	0	a	b	c
$f(x)$	0.93	0.74	0.82	0.55
$g(x)$	0.17	0.43	0.19	0.66

It is routine to verify that $\mathcal{E}_X := (X, f, g)$ is a $(3,2)$-fuzzy UP (BCC)-subalgebra of X.

(2) Let $X = \{0, a, b, c\}$ be a set with the binary operation "$*$" given in the following table:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	0	0	c
c	0	0	0	0

Then X is a UP (BCC)-algebra. Define a structure $\mathcal{E}_X := (X, f, g)$ on X by the table below:

X	0	a	b	c
$f(x)$	0.99	0.74	0.83	0.79
$g(x)$	0.11	0.65	0.38	0.41

It is routine to verify that $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy near UP (BCC)-filter of X.

(3) Let $X = \{0, a, b, c\}$ be a set with the binary operation "$*$" given in the following table:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	b
b	0	a	0	b
c	0	a	0	0

Then X is a UP (BCC)-algebra. Define a structure $\mathcal{E}_X := (X, f, g)$ on X by the table below:

X	0	a	b	c
$f(x)$	0.91	0.58	0.26	0.26
$g(x)$	0.19	0.48	0.58	0.58

It is routine to verify that $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy UP (BCC)-filter of X.

Proposition 3.3. *Every $(3, 2)$-fuzzy UP (BCC)-subalgebra $\mathcal{E}_X := (X, f, g)$ of X satisfies the condition (26).*

Proof. It is straightforward by the combination of (6) and (25). □

We provide conditions for a structure to be a $(3, 2)$-fuzzy UP (BCC)-subalgebra.

Theorem 3.4. *If a structure $\mathcal{E}_X := (X, f, g)$ satisfies:*

$$(29) \qquad (\forall x, y, z \in X)\left(z * x = 0 \Rightarrow \begin{cases} f^3(x * y) \geq \min\{f^3(y), f^3(z)\} \\ g^2(x * y) \leq \max\{g^2(y), g^2(z)\} \end{cases} \right),$$

then $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy UP (BCC)-subalgebra of X.

Proof. Since $x * x = 0$ for all $x \in X$ by (6), it follows from (29) that $f^3(x * y) \geq \min\{f^3(x), f^3(y)\}$ and $g^2(x * y) \leq \max\{g^2(x), g^2(y)\}$ for all $x, y \in X$. Therefore, $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy UP (BCC)-subalgebra of X. □

Corollary 3.5. *If a structure $\mathcal{E}_X := (X, f, g)$ satisfies the condition (29), then it satisfies the condition (26).*

Corollary 3.6. *If a structure* $\mathcal{E}_X := (X, f, g)$ *satisfies:*

$$(30) \qquad (\forall x, y, z \in X) \left(\begin{array}{l} f^3(x * y) \geq \min\{f^3(y), f^3(z)\} \\ g^2(x * y) \leq \max\{g^2(y), g^2(z)\} \end{array} \right),$$

then $\mathcal{E}_X := (X, f, g)$ *is a* $(3,2)$*-fuzzy UP (BCC)-subalgebra of* X*.*

Proof. It is clear that if $\mathcal{E}_X := (X, f, g)$ satisfies (30), then it also satisfies (29). Hence, $\mathcal{E}_X := (X, f, g)$ is a $(3,2)$-fuzzy UP (BCC)-subalgebra of X by Theorem 3.4. □

The following example shows that there exists a structure $\mathcal{E}_X := (X, f, g)$ on X in which the condition (26) holds but the condition (29) does not hold.

Example 3.7. *In Example 3.2(3), define a structure* $\mathcal{E}_X := (X, f, g)$ *on* X *by the table below:*

X	0	a	b	c
$f(x)$	0.99	0.74	0.79	0.83
$g(x)$	0.17	0.45	0.66	0.19

Then $\mathcal{E}_X := (X, f, g)$ *satisfies the condition* (26)*. But the condition* (29) *does not hold because of* $c * b = 0$ *but*

$$f^3(b * c) = f^3(b) = 0.493039 \ngeq 0.571787 = f^3(c) = \min\{f^3(c), f^3(c)\}$$

also

$$g^2(b * c) = g^2(b) = 0.4356 \nleq 0.0361 = g^2(c) = \max\{g^2(c), g^2(c)\}.$$

The following example shows that there exists a structure $\mathcal{E}_X := (X, f, g)$ in which the condition (29) holds but the condition (30) does not hold.

Example 3.8. *Let* $X = \{0, a_1, a_2, a_3\}$ *be a set with the binary operation "$*$" given in the following table:*

$*$	0	a_1	a_2	a_3
0	0	a_1	a_2	a_3
a_1	0	0	a_3	a_3
a_2	0	a_1	0	0
a_3	0	a_1	a_2	0

Then X *is a UP (BCC)-algebra. Define a structure* $\mathcal{E}_X := (X, f, g)$ *on* X *by the table below:*

X	0	a_1	a_2	a_3
$f(x)$	0.87	0.18	0.36	0.26
$g(x)$	0.42	0.92	0.58	0.87

It is routine to verify that $\mathcal{E}_X := (X, f, g)$ *satisfies the condition* (29). *The calculation below* $f^3(a_1{*}a_2) = f^3(a_3) = 0.017576 < 0.046656 = \min\{f^3(0), f^3(a_2)\}$ *and/or* $g^2(a_1{*}a_2) = g^2(a_3) = 0.7569 > 0.3364 = \max\{g^2(0), g^2(a_2)\}$ *shows that* $\mathcal{E}_X := (X, f, g)$ *does not satisfy the condition* (30).

We provide conditions for a structure to be a $(3, 2)$-fuzzy near UP (BCC)-filter.

Theorem 3.9. *If a structure* $\mathcal{E}_X := (X, f, g)$ *on* X *satisfies:*

$$(\forall x, y \in X)(x * y = 0 \;\Rightarrow\; f^3(x) \leq f^3(y),\, g^2(x) \geq g^2(y)), \tag{31}$$

then it is a $(3, 2)$*-fuzzy near UP (BCC)-filter of* X*.*

Proof. Let $\mathcal{E}_X := (X, f, g)$ be a structure on X that satisfies the condition (31). The condition (26) is induced by the combination of (3) and (31). Since $y * (x * y) = 0$ for all $x, y \in X$, it follows from (31) that $f^3(y) \leq f^3(x * y)$ and $g^2(y) \geq g^2(x * y)$ for all $x, y \in X$. Hence, $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy near UP (BCC)-filter of X. □

The following example shows that the converse of Theorem 3.9 is not true.

Example 3.10. *Consider* $(3, 2)$*-fuzzy near UP (BCC)-filter* $\mathcal{E}_X := (X, f, g)$ *of* X *which is given in Example 3.2(2). Since* $b * a = 0$, $f^3(b) = 0.83^3 \nleq 0.74^3 = f^3(a)$ *and* $g^2(b) = 0.38^2 \ngeq 0.65^2 = g^2(a)$, *we confirm that* $\mathcal{E}_X := (X, f, g)$ *does not satisfy the condition* (31).

We discuss relationship among a $(3, 2)$-fuzzy UP (BCC)-subalgebra, a $(3, 2)$-fuzzy near UP (BCC)-filter and a $(3, 2)$-fuzzy UP (BCC)-filter.

Theorem 3.11. *Every* $(3, 2)$*-fuzzy near UP (BCC)-filter is a* $(3, 2)$*-fuzzy UP (BCC)-subalgebra.*

Proof. It is straightforward by definitions. □

By the combination of Proposition 3.3 and Theorem 3.11, we know that every $(3, 2)$-fuzzy near UP (BCC)-filter $\mathcal{E}_X := (X, f, g)$ of X satisfies the condition (26).

In Example 3.2(1), the $(3, 2)$-fuzzy UP (BCC)-subalgebra is not a $(3, 2)$-fuzzy near UP (BCC)-filter because of $f^3(c * b) = f^3(a) = 0.74^3 = 0.405224 \ngeq 0.551368 = 0.82^3 = f^3(b)$ and/or $g^2(c * b) = g^2(a) = 0.43^2 \nleq 0.0.19^2 = g^2(b)$. This shows that the converse of Theorem 3.11 is not valid.

We provide condition(s) for a $(3, 2)$-fuzzy UP (BCC)-subalgebra to be a $(3, 2)$-fuzzy near UP (BCC)-filter.

Theorem 3.12. *If a* $(3, 2)$*-fuzzy UP (BCC)-subalgebra* $\mathcal{E}_X := (X, f, g)$ *of* X *satisfies* $f^3(x) \geq f^3(y)$ *and* $g^2(x) \leq g^2(y)$ *for all* $x, y \in X$ *with* $x * y \neq 0$, *then it is a* $(3, 2)$*-fuzzy near UP (BCC)-filter of* X*.*

Proof. Let $x, y \in X$. If $x * y \neq 0$, then $f^3(x * y) \geq \min\{f^3(x), f^3(y)\} = f^3(y)$ and $g^2(x * y) \leq \max\{g^2(x), g^2(y)\} = g^2(y)$ by (25) and the given assumption. If $x * y = 0$, then $f^3(x * y) = f^3(0) \geq f^3(y)$ and $g^2(x * y) = g^2(0) \leq g^2(y)$ by Proposition 3.3. Therefore, $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy near UP (BCC)-filter of X. □

Proposition 3.13. *Every* $(3, 2)$*-fuzzy UP (BCC)-filter* $\mathcal{E}_X := (X, f, g)$ *of* X *satisfies the condition* (31).

Proof. Let $x, y \in X$ be such that $x * y = 0$. Then

$$f^3(y) \geq \min\{f^3(x * y), f^3(x)\} = \min\{f^3(0), f^3(x)\} = f^3(x)$$

and

$$g^2(y) \leq \max\{g^2(x * y), g^2(x)\} = \max\{g^2(0), g^2(x)\} = g^2(x)$$

by (26) and (27). □

The combination of (10) and Proposition 3.13 leads to the following corollary.

Corollary 3.14. *Every* $(3, 2)$*-fuzzy UP (BCC)-filter* $\mathcal{E}_X := (X, f, g)$ *of* X *satisfies:*

$$(\forall x, y \in X)(f^3(y) \leq f^3(x * y),\ g^2(y) \geq g^2(x * y)). \tag{32}$$

Theorem 3.15. *If a structure* $\mathcal{E}_X := (X, f, g)$ *on* X *satisfies the condition* (31)*, then it is a* $(3, 2)$*-fuzzy UP (BCC)-subalgebra of* X*.*

Proof. Assume that a structure $\mathcal{E}_X := (X, f, g)$ satisfies the condition (31). By the combination of (10) and (31), we know that $\mathcal{E}_X := (X, f, g)$ satisfies the condition (30). Hence, $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy UP (BCC)-subalgebra of X by Corollary 3.6. □

Theorem 3.16. *Every* $(3, 2)$*-fuzzy UP (BCC)-filter is a* $(3, 2)$*-fuzzy near UP (BCC)-filter.*

Proof. Let $\mathcal{E}_X := (X, f, g)$ be a $(3, 2)$-fuzzy UP (BCC)-filter of X. For every $x, y \in X$, we have

$$f^3((x * y) \geq \min\{f^3(y * (x * y)), f^3(y)\} = \min\{f^3(0), f^3(y)\} = f^3(y)$$

and

$$g^2((x * y) \leq \max\{g^2(y * (x * y)), g^2(y)\} = \max\{g^2(0), g^2(y)\} = g^2(y).$$

Hence, $\mathcal{E}_X := (X, f, g)$ is a $(3, 2)$-fuzzy near UP (BCC)-filter of X. □

Corollary 3.17. *Every* $(3, 2)$*-fuzzy UP (BCC)-filter is a* $(3, 2)$*-fuzzy UP (BCC)-subalgebra.*

The converse of Theorem 3.16 may not be true. In fact, the $(3,2)$-fuzzy near UP (BCC)-filter $\mathcal{E}_X := (X, f, g)$ in Example 3.2(2) is not a $(3,2)$-fuzzy UP (BCC)-filter of X since

$$f^3(a) = 0.74^3 \not\geq 0.79^3 = \min\{f^3(c * a), f^3(c)\}$$

and/or

$$g^2(a) = 0.65^2 \not\leq 0.41^2 = \max\{g^2(c * a), g^2(c)\}.$$

We explore the conditions under which the converse of Theorem 3.16 is established.

Theorem 3.18. *Let* $\mathcal{E}_X := (X, f, g)$ *be a* $(3,2)$*-fuzzy near UP (BCC)-filter of* X*. If it satisfies:*

$$(\forall x, y \in X)(f^3(y) \geq f^3(x * y),\ g^2(y) \leq g^2(x * y)), \tag{33}$$

then $\mathcal{E}_X := (X, f, g)$ *is a* $(3,2)$*-fuzzy UP (BCC)-filter of* X*.*

Proof. Let $\mathcal{E}_X := (X, f, g)$ be a $(3,2)$-fuzzy near UP (BCC)-filter of X that satisfies the condition (33). The condition (26) is induced by Proposition 3.3 and Theoem 3.11. By the combination of (28) and (33), we have

$$f^3(y) \geq \min\{f^3(x), f^3(y)\} \geq \min\{f^3(x), f^3(x * y)\}$$

and $g^2(y) \leq \max\{g^2(x), g^2(y)\} \leq \max\{g^2(x), g^2(x * y)\}$. Therefore, $\mathcal{E}_X := (X, f, g)$ is a $(3,2)$-fuzzy UP (BCC)-filter of X. □

The theorem below is a characterization of a $(3,2)$-fuzzy UP (BCC)-filter.

Theorem 3.19. *A structure* $\mathcal{E}_X := (X, f, g)$ *on* X *is a* $(3,2)$*-fuzzy UP (BCC)-filter of* X *if and only if it satisfies:*

$$(\forall x, y, z \in X)\left(z * (x * y) = 0 \ \Rightarrow \left\{ \begin{array}{l} f^3(y) \geq \min\{f^3(x), f^3(z)\} \\ g^2(y) \leq \max\{g^2(x), g^2(z)\} \end{array} \right. \right). \tag{34}$$

Proof. Assume that $\mathcal{E}_X := (X, f, g)$ is a $(3,2)$-fuzzy UP (BCC)-filter of X and let $x, y, z \in X$ be such that $z * (x * y) = 0$. Then

$$f^3(x * y) \geq \min\{f^3(z * (x * y)), f^3(z)\} = \min\{f^3(0), f^3(z)\} = f^3(z)$$

and $g^2(x*y) \leq \max\{g^2(z*(x*y)), g^2(z)\} = \max\{g^2(0), \mathcal{E}^2(z)\} = g^2(z)$. Hence,

$$f^3(y) \geq \min\{f^3(x * y), f^3(x)\} \geq \min\{f^3(x), f^3(z)\}$$

and

$$g^2(y) \leq \max\{g^2(x * y), g^2(x)\} \leq \max\{g^2(x), g^2(z)\}.$$

Conversely, suppose that a structure $\mathcal{E}_X := (X, f, g)$ on X satisfies the condition (34). Let $x, y \in X$. Since $x * (x * 0) = 0$ by (3), we have $f^3(0) \geq \min\{f^3(x), f^3(x)\} = f^3(x)$ and $g^2(0) \leq \max\{g^2(x), g^2(x)\} = g^2(x)$. Since $(x * y) * (x * y) = 0$ by (6), we get $f^3(y) \geq \min\{f^3(x * y), f^3(x)\}$ and $g^2(y) \leq \max\{g^2(x * y), g^2(x)\}$. Therefore, $\mathcal{E}_X := (X, f, g)$ is a $(3,2)$-fuzzy UP (BCC)-filter of X. □

It is routine to verify that if a structure $\mathcal{E}_X := (X, f, g)$ on X satisfies (34), then it also satisfies the condition (31). Hence, we have the following corollary by Theorems 3.9 and 3.19.

Corollary 3.20. *If a structure $\mathcal{E}_X := (X, f, g)$ on X satisfies the condition (34), then it is a $(3, 2)$-fuzzy near UP (BCC)-filter of X.*

Theorem 3.21. *Given a fuzzy set f on X, consider the induced structure $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ on X where*

$$\tilde{g} : X \to [0, 1], \ x \mapsto 1 - f(x). \tag{35}$$

Then the induced structure $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is a $(3, 2)$-fuzzy set on X, and if it is a $(3, 2)$-fuzzy UP (BCC)-subalgebra (resp., $(3, 2)$-fuzzy near UP (BCC)-filter, $(3, 2)$-fuzzy UP (BCC)-filter) of X, then f is a fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP-filter) of X.

Proof. For every $x \in X$, we have

$$0 \leq f^3(x) + \tilde{g}^2(x) = f^3(x) + (1 - f(x))^2 \leq f^3(x) + 1 - f^3(x) = 1,$$

and so $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is a $(3, 2)$-fuzzy set on X. Assume that $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is a $(3, 2)$-fuzzy UP (BCC)-filter of X. Then $f^3(0) \geq f^3(x)$ and $f^3(y) \geq \min\{f^3(x * y), f^3(x)\}$ for all $x, y \in X$. Hence, $f(0) \geq f(x)$ for all $x \in X$. If $\min\{f^3(x * y), f^3(x)\} = f^3(x * y)$ or $\min\{f^3(x * y), f^3(x)\} = f^3(x)$, then $f(y) \geq f(x*y)$ or $f(y) \geq f(x)$. Thus $f(y) \geq \min\{f(x*y), f(x)\}$. Therefore, f is a fuzzy UP-filter of X. By the similar way, we can check that if $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is a $(3, 2)$-fuzzy UP (BCC)-subalgebra (resp., $(3, 2)$-fuzzy near UP (BCC)-filter) of X, then f is a fuzzy UP-subalgebra (resp., fuzzy near UP-filter) of X. □

We now discuss the relationship between an intuitionistic fuzzy UP-subalgebra and a $(3, 2)$-fuzzy UP (BCC)-subalgebra.

Theorem 3.22. *Every intuitionistic fuzzy UP-subalgebra is a $(3, 2)$-fuzzy UP (BCC)-subalgebra.*

Proof. Let $\mathcal{E} := (X, f, g)$ be an intuitionistic fuzzy UP-subalgebra of X. Then $f(x*y) \geq \min\{f(x), f(y)\}$ and $g(x*y) \leq \max\{g(x), g(y)\}$ for all $x, y \in X$. We consider the following cases:

(1) $f(x) \geq f(y)$ and $g(x) \geq g(y)$,
(2) $f(x) \geq f(y)$ and $g(x) < g(y)$,
(3) $f(x) < f(y)$ and $g(x) \geq g(y)$,
(4) $f(x) < f(y)$ and $g(x) < g(y)$.

The first case implies $f^3(x) \geq f^3(y)$ and $g^2(x) \geq g^2(y)$, and so

$$\begin{aligned} f^3(x * y) &= (f(x * y))^3 \geq (\min\{f(x), f(y)\})^3 \\ &= (f(y))^3 = f^3(y) = \min\{f^3(x), f^3(y)\} \end{aligned}$$

and

$$\begin{aligned} g^2(x*y) &= (g(x*y))^2 \leq (\max\{g(x), g(y)\})^2 \\ &= (g(x))^2 = g^2(x) = \max\{g^2(x), g^2(y)\} \end{aligned}$$

for all $x, y \in X$. The case 4 implies that $f^3(x) < f^3(y)$ and $g^2(x) < g^2(y)$, and so

$$\begin{aligned} f^3(x*y) &= (f(x*y))^3 \geq (\min\{f(x), f(y)\})^3 \\ &= (f(x))^3 = f^3(x) = \min\{f^3(x), f^3(y)\} \end{aligned}$$

and

$$\begin{aligned} g^2(x*y) &= (g(x*y))^2 \leq (\max\{g(x), g(y)\})^2 \\ &= (g(y))^2 = g^2(y) = \max\{g^2(x), g^2(y)\} \end{aligned}$$

for all $x, y \in X$. In the rest of the cases, we can check that the condition (25) is true in the same way. Hence, $\mathcal{E} := (X, f, g)$ is an $(3,2)$-fuzzy UP (BCC)-subalgebra of X. □

The converse of Theorem 3.22 may not be true. In fact, the $(3,2)$-fuzzy UP (BCC)-subalgebra $\mathcal{E} := (X, f, g)$ of X in Example 3.2(1) is not an intuitionistic fuzzy UP-subalgebra of X because of $f(a) + g(a) = 1.17 \not\leq 1$.

It is clear that if f is a fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP-filter) of X, then the induced structure $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is an intuitionistic fuzzy UP-subalgebra (resp., intuitionistic fuzzy near UP-filter, intuitionistic fuzzy UP-filter) of X. Hence, we have the following corollary.

Corollary 3.23. *If f is a fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP-filter) of X, then the induced structure $\tilde{\mathcal{E}}_X := (X, f, \tilde{g})$ is a $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter, $(3,2)$-fuzzy UP (BCC)-filter) of X.*

4. Conclusion

The idea of intuitionistic fuzzy sets suggested by Atanassov is one of the extension of fuzzy sets with better applicability in medical diagnosis, optimization problems, and multicriteria decision making, etc. Ibrahim introduced the notion of $(3,2)$-fuzzy set as another extension of intuitionistic fuzzy sets. In this paper, we have applied the $(3,2)$-fuzzy set to the UP (BCC)-subalgebras and UP (BCC)-filters of UP (BCC)-algebras. We have introduced the concept of $(3,2)$-fuzzy UP (BCC)-subalgebra, $(3,2)$-fuzzy near UP (BCC)-filter and $(3,2)$-fuzzy UP (BCC)-filter in UP (BCC)-algebras, and investigated several properties, including their relations. We have given the conditions under which the $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter) can be the $(3,2)$-fuzzy near UP (BCC)-filter (resp., $(3,2)$-fuzzy UP (BCC)-filter). We have provided some characterizations of $(3,2)$-fuzzy UP (BCC)-filter and discussed the relationship between intuitionistic fuzzy

UP-subalgebra and $(3,2)$-fuzzy UP (BCC)-subalgebra. We have used fuzzy UP-subalgebra (resp., fuzzy near UP-filter, fuzzy UP (BCC)-filter) to create a $(3,2)$-fuzzy UP (BCC)-subalgebra (resp., $(3,2)$-fuzzy near UP (BCC)-filter, $(3,2)$-fuzzy UP (BCC)-filter).

In future work, we will apply $(3,2)$-fuzzy sets to other algebraic structures such as MV-algebras, equality algebras, hoops, EQ-algebras, etc. Also, we will try to study soft and rough set theory based on $(3,2)$-fuzzy algebraic structures. We will explore the possibility of using the ideas and results of this paper for decision-making theory or medical diagnosis.

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