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COMPARISON BETWEEN MATHEMATICAL PROBLEM-SOLVING APPROACH UNDER IRANIAN AND IRAQ TEACHERS' VIEWS

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ABSTRACT. Being mainly a process of knowledge transmission, mathematics education evolves during time in accordance with the strong assumptions and beliefs which are considered as parts of the mathematics teaching profession. This suggests that explaining the problem-solving process, transmitting the clear and flawless information, and showing the problem-solving procedures, were parts of the role the mathematics teachers have. The main purpose of this study was to compare the mathematical teaching experiences based on the problem-solving approach among the Iranian and Iraq mathematics educators. Through survey method, views of secondary teachers of mathematics are studied. It is used of questionnaire that is proposed by Matlala's (2015). The validity and reliability has been proved by researcher using Cronbach's alpha method with a value more 0/89 This questionnaire was designed with the purpose of identifying challenges and opportunities that every individual encounter with in the way of using a problem-solving approach to facilitate mathematics learning. The statistical population of the study included all the secondary school math teachers in Iran and Iraq. Using the simple random sampling method, 16 secondary school math teachers from the Republic of Iraq (from its capital: Kurdistan) and 14 secondary school math teachers from the Islamic Republic of Iran (from its capital: Tehran) were selected. Through the use of an electronic questionnaire, was sent to the in-service teachers during the school year 2018-2019. The findings indicated that Iranian and Iraq teachers' view regard to the implementation of problem solving procedure were positive and they have applied problem solving procedure in their math classes.

 ${\it Keywords} :$ Problem solving, Mathematics, Teaching method, Teachers' views.

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1. Introduction

The goals of mathematics education in the Arab States are more or less similar. The third conference of the Arab Ministers of Education' approved, among

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other things, the goals of mathematics education in the Arab States. These goals centred on: - attaining arithmetical literacy for all, and mathematical literacy and competence for the majority; - preparing students for subsequent study or professions; and - developing desirable habits and favourable attitudes.

The goals of mathematics education include also the development of habits and attitudes such as: - being accurate and organized; - acquiring an interest in mathematics; - building self-confidence; and - promoting sound moral and social attitudes. The mathematics curriculum in the Arab States has to face the challenges posed by the new types of schools that are being tried in some Arab countries. Egypt is experimenting with the comprehensive school, and with the polytechnic school. Several other Arab countries have started similar experiments. The Sudan is experimenting with the 'Integrated Rural Education Centre', which is a new type of elementary school, geared towards rural education and providing also informal adult education. New mathematics curricula of appropriate content, organization and approach have to be developed to serve the goals of these new types of school [23].

Academic achievement in mathematics is the most important problem in education system of any country in particular at secondary level. Teachers of mathematics have faced to the kind of difficulties in the field of teaching mathematics, geometry and algebra in their classes. This procedure has many challenges. Reaching to the common understanding of the problem-solving concept is difficult, since everyone defines it differently. Mathematical thinking is a mathematical process that includes five aspects, one of which is mathematical problem solving Heleni et al [5]. Mathematical problem-solving ability is a basic ability in learning mathematics. This shows that the ability to solve mathematical problems is an ability that must be possessed by students and is one of the factors that determine student learning outcomes in mathematics Guswinda et al [4]. According to Lesh & Doerr [11] we can also observe problem solving as the process of moving from givens to goals without having a clear way ahead. According to Stacy [26] a successful and an effective mathematical problem-solving rest on having reasoning skills, great mathematical knowledge, abilities for peer and group work, personal characteristics such as confidence, persistence, and organization, and finally the communication abilities. According to Panaoura [17] helping students in approaching the problems and employing appropriate mathematical problem-solving tools, cause the mathematics teachers to frequently face with some challenges. Moreover, teachers should gain more flexible and deeper subject matter knowledge compared to the knowledge needed to follow the guidelines or a fixed lesson plan proposed in the textbook or a teacher's guidebook. As Furthermore, as Artzt et al [2] indicated, teacher educators lack the ability of preparing the teachers to teach in a way according to new ideas concerning the nature of mathematics and learning. Teachers themselves mostly lack the problem-solving abilities which they are supposed to model for their students, and they have not even shown

them in their class presentations, which is considered as another challenge they mostly face. Through spending time with those who are able to model proper problem-solving skills, teachers can achieve success in teaching such skills and the students can improve their problem-solving ability to turn to proficient problem solvers. The other challenge that finally the teachers face in teaching problem solving, is that they might be required to modify some of their main beliefs about the components of mathematical literacy regarding to the following questions: do students show mathematical literacy through the purposeful use of mathematical information to make connections between mathematical ideas? And or when the student's memories computation algorithms and facts, do they show mathematical literacy?

Examples of similar studies in this area are as follows: In a study on investigating the effectiveness of teaching Polya's problem solving techniques on the mathematical self-efficacy of the sixth-grade elementary school students, Vaghar et al [27] showed that teaching Polya's problem solving techniques has a positive effect on students' self-efficacy. Phan et al [18] studied the use of cognitive load theory in mathematical problem solving. They indicated that cognitive load imposition plays a key role in t instructional programs for better learning. They explored varying efficiencies of different instructional procedures, taking into consideration the potency of cognitive load imposition.

Mo'meni Mahmoei et al [14] in a study on the effectiveness of teaching Polya's problem solving strategies on the self-concept and academic achievement of elementary students, showed that teaching problem-solving strategies leads to academic achievement and self-concept improvement in students. In a study on the mathematical problem-solving skills of the pre-service teachers and their abilities in solving daily problems, Aljaberi and Gheith [1], based on the Polya's problem solving steps, showed the students' weaknesses in their mathematical problem-solving skills, with statistically no significant differences between learners in solving mathematical problems. On the other hand, significant differences were found in learners' performance in solving mathematical problems concerning the mathematical subject matters, and on the side of algebra and measurements; moreover, significant differences were also found in students' mathematical problem-solving skills regarding high-school and academic year stream, however there were no correlation between learners' mathematical problem-solving skills and their daily problem-solving abilities. In a study entitled "Defining Mathematical Problems and Problem Solving: Prospective Primary Teachers' Beliefs in Cyprus and England", Constantions and Paul [28] showed the differences and similarities in interpreting both concepts of problems and problem solving in mathematical domain by the prospective teachers in England and Cyprus; which revealed that not only they have culturally-based beliefs, but also that they interpret the notions of "problem" and "problem solving" in different ways, cross-culturally. According to what was mentioned above, the researchers are seeking to study the approaches towards solving mathematical problems in schools. Since the curriculums and educational planning vary in each country and in each educational system, this study seeks to compare the problem-solving approach in teaching and learning the mathematical concepts in classroom. Given the fact that during the recent years, Iraq has had a dramatic growth in mathematical education, the researchers seek to investigate on such an important approach between Iran and Iraq. Therefore, the main purpose of this study was to compare the mathematical teaching experiences based on the problem-solving approach among the Iranian and Iraq mathematics educators.

2. Problem-solving framework

- 2.1. Polya's problem-solving framework. Polya [19], considers four phases for problem solving. These phases or steps consist of "understanding the problem", "devising a plan", "carrying out the plan" and "looking back". Advocates of Polya's framework use a step-by-step instruction in their teaching practice: the first step which is called Understanding the problem is raised based on the Polya [19] belief which considers the understanding of the problem's verbal statement necessary to be able to understand the problem itself. Students must be able to repeat the statement of the problem via their own words. Based on the beliefs of [19] the students are supposed to be helped by their teachers in formalizing and establishing their ideas. Teachers should have only a little interference in the process of finding problem solving strategies. Too much interference from the side of the teachers is harmful for it takes the opportunity of working on the activity and finding the solution from the students and this way, the goal of the activity will not be reached.
- 2.2. Schoenfeld's problem-solving framework. Schoenfeld's framework is grounded on Gestaltism learning theory which was based on cognitive science perspective and developed from it. As Schoenfeld [21] stated, Gestaltists valued rich mental structures and believed that the goal of teaching must be to help the students to develop these structures. Moreover, Schoenfeld [22] introduced categories like belief systems, heuristics, and control which are necessary for the time of working on mathematical problems:

Resources: Schoenfeld [21] defined resources as skills, procedures, tools, and facts that the problem solvers potentially have access to them. Resources refer to the mathematical procedures and knowledge that the learner uses in solving a specific problem.

Heuristics: Heuristics stand for the rules, techniques, and strategies that help successful problem solving, and recommendations that help the learners to understand the problem better and make progress in reaching to the solution. Larson [10] likewise asserts that heuristics are tactics or strategies of solving problems like mathematical problems. To name some of these strategies, we can refer to working backwards, introducing relevant notations, testing and verifying procedures, exploiting related problems, arguing by contradiction,

reformulating problems, drawing figures, and working forward from available data.

Control: Control refers to the way of using the information by the learners while solving the problems. Control concentrates on decision makings concerning what to do in solving a problem which might lead one to take an attempt to solve that problem or stop him from doing that.

Belief systems: belief systems refer to the individual's worldview in the field of mathematics; that is the perspective and viewpoint that one uses in approaching mathematics and working on mathematical problems. Belief systems contain the beliefs about the environment, the topic, mathematics, and the self. The individual's belief about mathematics has a great impact on the extent of time and energy he spends in working on the problem, the techniques he decides to use or avoid, and the way he decides to approach a problem. From the problem-solving frameworks introduced by Schoenfeld and Polya, it can be inferred that instead of looking for a single correct answer, the students follow some steps including deducing the problem, gathering the proper information, evaluating the possible options and finally presenting the conclusions as the appropriate solution. These two frameworks in the domain of teaching and learning which have role in enhancing the deep mathematical understanding of the students, gave an integrated view of problem solving to the researcher, even though Kilpatrick et al [8] claims that we cannot find any framework that completely undertakes all aspects of mathematical knowledge, facility, competence, and expertise The mid-20th century proposed a modern view towards the mathematical problem-solving teaching and learning. Particularly, Polya [19] in his books on the importance of teaching mathematical problem solving at school, two of which were entitled "Mathematical Discovery" and "How to Solve It", emphasized on this issue. Nowadays, as we are in the 21st century, most of the researchers in the field of mathematics education have shifted their focus to teaching through problem solving, which according to Stacy [26] is relatively attracted the interests of many researchers, making the study in this area advice, scarce, and yet plentiful.

3. Mathematical problem-solving approach

In this section, according to Polya's and Schoenfeld's problem-solving approach, the researchers have attempted to design and introduce an example of a problem-solving approach based on the Polya's and Schoenfeld's problem-solving process in a series of stages. These stages are as follows:

3.1. Stages for implementing the problem-solving approach understanding the problem-goal setting. The main purpose of this stage is to understand and comprehend the problem and to set the goal of mathematics problem. Teachers try to determine the main goals of mathematics problems through following the steps below:

Read the given problem or task, and recapitulate or restate it with your own

words: at this stage, the teachers should restate the mathematical problem with their own level of understanding and comprehension. This is an informal way of statement of problem.

Interpret, visualize, or simulate the situation: In this section, the teachers should visualize a mathematical problem based on their own level of comprehension. This visualization can be in the form of a sentence, figure or a diagram that expresses the status and the main purpose of the problem.

Find the related assumptions and the data and determine the symbols: Teachers should separately find the data and the main information in the mathematical problem, and find the particular terms or symbols needed in the problem. Draw a figure or a diagram to organize the certain data: Teachers should interpret a mathematical problem with respect to a figure being drawn based on the given information. Drawing a figure can illustrate the real situations of the math problem to the teachers. This figure is drawn by the teacher (personally). Identify what is needed to find out: Teachers should specify the problem's purpose in the mathematical problem with reliance on finding the original solution. Therefore, the necessary cases in the mathematical problem will be identified. Determine whether the sufficient information is provided or the additional information is needed? Are there any unnecessary and redundant information? The goals in some mathematical problems are specified in two sections of main and secondary. By implementing this step, teachers prioritize the required information for the main and the secondary goals. Some problems may have unnecessary goals.

If possible /if necessary, restate the problem to clarify it. In mathematical problems, teachers can restate the problem description in the simplest sentences as much as possible, which forms another formulation of the problem. What is needed is the restatement of the mathematical problem in a precise and purposeful form.

4. Introducing a design and a solution strategy for a math problem

The main goal of this stage is to determine a strategy to solve the mathematical problem. Teachers try to determine a design and a solution strategy for a math problem by following the steps below:

Find the key elements of the problem and focus on how to achieve them: after selecting and determining the important information of the problem, the teachers try to choose some methods and strategies and find the relationship between them. What is important is to focus on the core components, not the key components of the mathematical problem.

Simplify the problem (choose smaller numbers, change the assumptions, consider special cases): Some math problems may contain intangible numbers and terms. Teachers can modulate large components and numbers in the problem to make finding a mathematical problem-solving strategy easy.

Identify and validate a pattern with a conscious guess: teachers can do pattern making for a math problem. Finding the pattern and the constructive relationship between the main components of the mathematical problem is important, so the teachers should find a pattern for some complex mathematical problems, in addition to redefining or restating the math problem. This pattern can specify the main components of the problem more precisely.

Analyze the problem and try to identify a step-by-step approach: Teachers can provide a list of the main and secondary goals of the math problem, and then find the solution strategies, according to the priorities of the math problem.

Find a similar problem and try to use the same solution: Some math problems require the use of a similar problem which has already been solved. According to their experience, teachers can find a problem similar to the main problem, and generalize the solution of the similar problem to the main problem.

Choose a particular approach and try to master it, as much as you can: teachers can find a main strategy for solving a math problem and try to dominate or master the main solution, perhaps the same solution can be applied to similar mathematical problems.

Know where you are standing at and what the goal is and try to get closer to them by restating the current status or goal: after restating the problem status, teachers constantly try to identify the main status and goal of the problem, in order not to distract from the main goal.

4.1. Implementing the problem's design, reviewing the design and, if necessary, modifying it. The main purpose of this stage is to determine the problem's design and to modify it in a mathematical problem. Teachers try to determine the basic design of the math problem through the following steps: Record and explain the problem-solving steps: Teachers must record each of the solutions after restating the problem and following the previously mentioned stages, and for choosing each solution, they must record their reasons for choosing them.

Identify the tools (methodology and techniques) needed for the solution: Some math problems might require specific methods to reach to the final answer. Such methods can be obtained through studying and consulting.

Check the work step-by-step: Teachers should examine and test each mathematical problem step-by-step through specifying the primary and secondary objectives, and by focusing on the solutions at each stage. Some steps along with their solutions might not be taken place in the correct way, hence interrupting the solution for the next steps.

If a problem design does not lead to a successful solution, evaluate it and find another more appropriate design: In this step, if some solutions and strategies are not effective at each step and in each part of the problem pattern, then the more efficient strategies and solutions must be used, so that the strategies and solutions of the problem's next steps won't face any trouble. The need for a step-by-step review shows the success or failure of the strategies.

4.2. Checking and generalizing the problem. The main purpose of this stage is to check and generalize the mathematical problem. Teachers try to review and generalize the mathematical problems through the following steps: Evaluate and review the result and check if it is understandable or not: at this stage, teachers can evaluate and criticize the effectiveness of each solution at each step of problem solving process, i.e., if we do not use this solution, what other solution will be more efficient and why this solution has been used?

Find a way to check the solution with an independent path: Teachers should find a way to evaluate their solutions at each step of the problem pattern. This type of evaluation should be done individually at each step, so that the analysis of the solution will become more accurate.

Check the validity of the conclusion: after implementing and presenting the solution in each step of the problem and examining the main and secondary goals of the problem, you must test the main conclusion or the final answer to the problem. Teachers can examine this method in a retrospective review in accordance with Polya's method .

Write a brief and clear solution and evaluate the method: To achieve the final answer to a math problem, you can provide a practical, but a brief solution, so that the general form of the solution will be determined.

Find another solution to the problem: Teachers can find other solutions, similar to the problem steps, and by reliance on the main goal of the problem. They can also find the best and the most efficient solution from the proposed solutions.

Find the generalizations and the extensions: Some mathematical problems might need to add other meanings, in order to clarify the problem. Given the main goals and the essential components of the problem, teachers can add more complete restatements to the problem, in order to clarify the main status of the problem.

Create new questions. Create a new problem by changing the data or assumptions of the previous problem: Teachers can, as far as possible, design or create the newer and more efficient problems through changing the main components of the problem and by relying on the main goal of the problem, to better clarify the main problem.

In addition to the emphasize on the stages of problem solving (internal dialogue) and heuristic methods, focus on the various and multiple representations, as well as the effective and metacognitive elements of problem-solving: teachers can reach the metacognitive level, through self-questioning and by relying on the control stage, as one of the Schoenfeld's problem solving steps; in other words, through self-questioning, teachers can review the main goals and the main components of the problem, isolate the inefficient components of the problem, and find the main goals through restating by relying on the mathematical problem pattern making. Teachers can then determine the efficient methods for solving the problem in each step.

4.3. Evaluation of the implementation method of the "problem solving" stages. Problem solving stages: All problem-solving stages were presented using various representations such as objective activities, figures and symbolic manipulations, through exploring and studying several problems. Breaking down the problems into their stages and the way of creating new problems during the problem-solving process were practiced. The type of questions that individuals should ask from themselves during the problem-solving process, that if the teachers know, they can understand the problem-solving paths better, were practiced. Teachers practiced the recognition of new problems, recording the relevant information and connections in different formats, and controlling and evaluating the strategies.

Test, Samples (Examples), and counterexample: It was tried to emphasize the importance of the test in the problem-solving process. Teachers guessed and controlled the method, and demonstrated the accuracy of the method by using examples or showed the inaccuracy of the suggested methods with counterexamples. Teachers were also given the opportunity to recognize whether their hypothesis were proved or confirmed, and also the opportunity to practice their reasoning. Drawing (drawing figure): Drawing and expression of the visual representations (figures and diagrams) is very useful when determining the main components of the problem. The visual representations were implemented to help solving the problems which can be solved by the method of drawing diagrams and figures.

Problem set by asking "What if ... not ...?": We offered a problem-solving strategy by asking the question "What if ... not ...?". By changing the data or the problem assumptions, new problems were created, and the problem-solving steps were implemented on each of them by the constructor. As a result, a small project started to work and implemented in the relevant area.

A Model - several multiple representations: Several representations of the problems with common ground were presented. This helped teachers to identify the similarities in new problems, and to apply the problem-solving methods that they knew. Verbal problems were described in mathematical operations. Based on the presented problems, teachers were required to collect problems for a particular model.

5. Mathematical Knowledge for Teaching (MKT)

The contributions of Shulman [24] on teacher knowledge reveal the importance of content knowledge for teaching, defined as "the amount and organization of knowledge per se in the mind of the teacher" (p.9), and pedagogical knowledge as "the particular form of content knowledge that embodies the aspect of content most germane to its teachability" (p.9). This has given way to the study and development of other models derived from Shulman's [24, 25] in this area specific for mathematics teachers [12, p.31-33], showing that content knowledge is more effective in teaching when it is combined with pedagogical

knowledge [12, 16].

Ball et al [12] based on the knowledge framework proposed by Shulman (1986-1987), identified the essential elements of knowledge to teach mathematics and developed the mathematical knowledge for teaching (MKT) model, described as "the mathematical knowledge needed to carry out the work of teaching mathematics" Ball et al ([12], p.395).

Based on an analysis of teacher practices, Ball et al [12] determined the mathematical demands of teaching that would later make up the model's components, providing an empirical basis for the positive relationship that constitutes the pedagogical knowledge of teachers and the learning outcomes of students. Accordingly, the MKT model establishes a domain map of mathematical knowledge for teaching that considers content knowledge and pedagogical content knowledge.

Content knowledge is subdivided into three aspects: common content knowledge (CCK), which refers to the "mathematical knowledge and skill used in settings other than teaching" Ball et al (Ball et al [12], p.399), meaning it corresponds to the knowledge that can be achieved throughout the educational levels and that anyone who faces a mathematical task possesses; specialized content knowledge (SCK), which refers to "mathem.atical knowledge and skill unique to teaching Ball et al ([12], p.400), meaning knowledge that is specific to the teacher and that is used to engage in teaching tasks related to "how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems" Hill et al ([7], p.377-378); and knowledge of the mathematical horizon, which is defined as "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" Ball et al ([12], p.403). meaning the knowledge that allows the teacher to establish the way in which the mathematical contents are related to other contents of the curriculum.

A pedagogical knowledge of the content considers: the knowledge of the content and the students (KCS), which is defined as the "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" Ball et al ([12], p.375), i.e., it is the knowledge that the teacher has about the knowledge of the students, letting the teacher predict situations and anticipate the concerns, attitudes or difficulties of the students; knowledge of content and teaching (KCT), "combines knowledge about teaching and knowing about mathematics" Ball et al ([12], p.401), meaning knowledge that integrates specific mathematical knowledge and pedagogical and didactic aspects of the teaching processes involved in student learning; and, finally, knowledge of the curriculum, "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, and the variety of instructional materials available in relation to those programs" Ball et al ([12], p.391) meaning it refers to the orientations and approaches corresponding to

the study programs designed for each educational level in the area of mathematics, together with the instructional materials.

6. Mathematics education in singapore

Singapore is an island, with an area of 712.4 square metres. The population is approximately 5.4 million of which more than one million are foreigners working in the country. The GDP per capita as of November 2013 is Singapore Dollars \$90,166. The two largest budget items of the government expenditure are Defense and Education. Education for primary, secondary, and tertiary levels is mostly supported by the state. All institutions, private and public, must be registered with the Ministry of Education. English is the language of instruction in all public schools and all subjects are taught and examined in English except for the "Mother Tongue" language paper. While "Mother Tongue" generally refers to the first language internationally, in Singapore's education system it is used to refer to the second language as English is the first language. Education takes place in three stages: "Primary education", "Secondary education", and "Postsecondary education". Pupils begin with six years of primary school, which is made up of a four-year foundation stage and a two-year orientation stage. In the foundation stage the curriculum is focused on the development of English, the mother tongue, and mathematics. In the orientation stage pupils study the four standard subjects: English, the mother tongue, mathematics, and science. Secondary school lasts from four to five years, and is divided between "Special", "Express", "Normal (Academic)", and "Normal (Technical)" courses of study within each school, depending on a pupil's ability level. The school curriculum is comprehensive and pupils take subjects in Languages, Arts, Humanities and Sciences. Post-secondary education takes place from two to three years at Junior Colleges, polytechnics and institutes of technical education. For junior colleges, the curriculum comprise of specialized subjects and a contrasting subject for a broad based education. For polytechnics and institutes of technical education the curriculum is specialized and specific to the course of study the students are undergoing, for example, business studies, mass communication, engineering, etc. National examinations are standardized across all schools, with a test taken after each stage of school. After the first six years of education, pupils take the Primary School Leaving Examination, which determines their placement at secondary school. At the end of secondary education pupils take the General Certificate of Secondary Education examinations and at the end of pre-university education they take the General Certificate of Advanced Education examinations. This paper presents a historical perspective on developments that have shaped mathematics education in Singapore and evolution of school mathematics curricula. It also explores factors that may explain the achievement of Singapore students in mathematics.

Pupils sit a national examination called the Primary School Leaving Examination (PSLE) at the end of Primary six. The examination assesses pupils suitability for secondary education and places them in an appropriate secondary school course that matches their learning ability. Three Courses are available at the secondary school level. Pupils undergo four or five years of secondary education with different emphases.

- 1. Special Course a four-year course leading to the Singapore-Cambridge General Certificate of Education (GCE) 'O' level examination. In this course, pupils study their mother tongue at an advanced level, in addition to the usual humanities, mathematics and science subjects.
- 2. Express Course also a four-year course leading to the GCE 'O' level examination. In this course pupils study their mother tongue at an ordinary level and offer a curriculum similar to that in the Special course.
- 3. Normal Course a four-year course leading to the GCE 'N' level examination. A fifth year is available to pupils who do well in this examination to prepare for and take the GCE 'O' level examination. Pupils in this course follow either the Normal (Academic) or Normal (Technical) curriculum. In the N(A) curriculum, they will learn English, mother tongue, mathematics and a range of subjects similar to those in the Special and Express courses. In the N(T) course, pupils will learn English, mother tongue at a basic level emphasizing oral/aural competence and reading comprehension, mathematics, computer applications and subjects with a technical and practical bias such as technical studies [9].

As mathematics is a compulsory subject for pupils in school, the mathematics curriculum at the secondary school level is differentiated to cater to the needs and abilities of pupils in the different courses. Core mathematical concepts are common to all courses and the content for the Special Course is identical to the Express Course. The content for the Normal (Academic) Course is a subset of the content for Special/Express Course while that of the Normal (Technical) Course is a subset of the Normal (Academic) Course. For all the three courses most of the topics taught at the various year levels for mathematics are similar. However the depth to which they are taught at a particular year level differs. The following extract from the syllabuses Mullis et al [15] highlight the varying depth.

7. Singapore's performance in TIMSS and PISA

Singapore's participation in TIMSS and PISA Singapore participates in international studies such as Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) to benchmark the outcomes of schooling, viz-a-viz the education system against international standards. It also does so to learn from educational systems that are excelling, to update school curriculum and keep abreast of global advances and to contribute towards the development of excellence in education

internationally. To date Singapore has participated in Trends in International Mathematics and Science Study (TIMSS) in 1995, 1999, 2003, 2007 and 2011. The following table shows Singapore students' achievement in mathematics for TIMSS 1995, 1999, 2003, 2007 and 2011.

The education system in Singapore is dynamic and constantly evolving. Initiatives and policies are guided by research evidence, scans of other systems in the world and careful deliberations of leaders in education. Whatever the new initiative or policy may be the one thing that always "keep the house in order" is the TEACHER. Therefore it is vital that the development of teachers keep abreast of changes in the system. Finally, as stated by the Mckinsey report Barbar et al [3] the quality of an educational system cannot exceed the quality of its teachers.

8. Method & Materials

Survey method was used in this study. Survey research as part of the descriptive research is a data collection method in which a particular group of people are asked to answer to a number of specific questions. In the survey research, the first step is the definition of the problem. That is, to conduct the study, the researcher must define his objectives, and any question that he asks must be related to one of the goals or more. In the second step, the population should be properly defined and determined. In the next step, the data collection method should be generally determined, and then a number of individuals should be randomly selected from the population, as samples. The most commonly used tool in survey research is the questionnaire. Therefore, according to the subject matter and the objectives of the study, a standard questionnaire was used. This questionnaire was the adapted version of the questionnaire used in the [13] master's thesis at Stellenbosch University, South Africa. The questionnaire consisted of two main parts. The first part contains the demographic information of the Iranian and Iraq mathematics teachers and the second part contains 36 items with five Likert scales. The components of the questionnaire and the objectives consistent with these components are as follows:

- Investigating the "teaching method (TM)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Studying the "Planning and Preparing the Lesson Plan (PPLP)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Investigating the "Relationship with the Learner (RL)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Investigating the "teacher inquiry (TI)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Investigating the "homework assignments and activities (HAA)" in problem solving approach of Iranian and Iraq mathematical teachers;

- Investigating the "class discourse (CD)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Investigating the "post-teaching sessions (PTS)" in problem solving approach of Iranian and Iraq mathematical teachers;
- Investigating the "teaching materials (for studying and reviewing) (TMSR)" in problem solving approach of Iranian and Iraq mathematical teachers.

The validity and reliability of this questionnaire have been proved by Matlala with acceptable values, and its reliability has been proved by researcher using Cronbach's alpha method with a value more than 0.91 and 0.88. This questionnaire was designed with the purpose of identifying challenges and opportunities that every individual encounter with in the way of using a problem-solving approach to facilitate mathematics learning.

"Students' problem-solving performance was highly correlated with their problem posing performance." Compared to less successful problem solvers, good problem solvers generated problems that were more mathematical, and their problems were more mathematically complex .by example:

Find the sum of the first 10, 100, and 500 counting numbers.

Understand the Problem

Since counting numbers are the numbers $1, 2, 3, 4, \ldots$, the sum of the first 10 counting numbers would be $1+2+3+\ldots+8+9+10$. Similarly, the sum of the first 100 counting numbers would be $1+2+3+\ldots+98+99+100$ and the sum of the first 500 counting numbers would be $1+2+3+\ldots+498+499+500$.

Devise a Plan

Rather than solve three different problems, the "Use a Variable" strategy can be used to find a general method for computing the sum in all three situations. Thus, the sum of the first n counting numbers would be expressed as $1+2+3+\ldots+(n-2)+(n-1)+n$. The sum of these numbers can be found by noticing that the first number 1 added to the last number n is n-1, which is the same as (n-1)+2 and (n-2)+3. Adding up all such pairs can be done by adding up all of the numbers twice.

Carry Out the Plan

$$1+2+\ldots+(n-2)+(n-1)+n+n+(n-1)+(n-2)+\ldots+2+1$$

$$=(n+1)+(n+1)+\ldots+(n+1)+(n+1)+(n+1)$$
(1)
$$=n\cdot(n+1).$$

Since each number was added twice, the desired sum is obtained by dividing $n \cdot (n-1)$ by 2 which yields

(2)
$$1+2+3+\ldots+(n-2)+(n-1)+n=\frac{n\cdot(n+1)}{2}.$$

The numbers 10, 100, and 500 can now replace the variable n to find our desired solutions:

$$1 + 2 + 3 + \dots + 8 + 9 + 10 = \frac{10 \cdot (10 + 1)}{2} = 55,$$

$$1 + 2 + 3 + \dots + 98 + 99 + 100 = \frac{100 \cdot (101)}{2} = 5050,$$

$$(3) \qquad 1 + 2 + 3 + \dots + 498 + 499 + 500 = \frac{500 \cdot (501)}{2} = 125250.$$

Look Back

Since the method for solving this problem is quite unique could it be used to solve other similar looking problems like:

(4)
$$i. \ 3+6+9+\ldots+(3n-6)+(3n-3)+3n,$$
$$ii. \ 21+25+29+\ldots+113+117+121$$

- 2. Show that the sum of any five consecutive odd whole numbers has a factor of 5.
- 3. The measure of the largest angle of a triangle is nine times the measure of the smallest angle. The measure of the third angle is equal to the difference of the largest and the smallest. What are the measures of the angles? (Recall that the sum of the measures of the angles in a triangle is 180°.)

9. Participates

The statistical population of the study included all the secondary school math teachers in Iran and Iraq. Since it was not possible for the researchers to have access to the accurate number of these individuals, it was tried to select some of the experienced math teachers from the different regions of the capitals of both countries. In this regard, using the simple random sampling method, 16 secondary school math teachers from the Republic of Iraq (kurdistan) and 14 secondary school math teachers from the Islamic Republic of Iran (from its capital: Tehran) were selected. Through the use of an electronic questionnaire, a copy of which is active through the link:

https://www.survio.com/survey/d/C0F6A1S4E5E4X4F5E, was sent to the inservice teachers during the school year 2018-2019. After responding to the questionnaire, the participants submitted their responses.

10. Findings

In this section, researchers have collected all data regard to teachers' views at secondary for teaching mathematics. After two months, all responses are collected. Descriptive statistics shown in Tables (1) to (8):

Table 1. Frequency of gender

	Iranian T	eachers	Iraq Teachers			
	Frequency	Percent	Frequency	Percent		
Male	4	28/60	10	62/5		
Femal	10	71/40	6	37/5		
Total	14	100	16	100		

The results of Table (1) indicated that the numbers of Iranian females $(62 \cdot 50\%)$ and Iraq males $(71 \cdot 40\%)$ have the most frequencies than other.

Table 2. Frequency of highest educational level

	Iranian T	eachers	Iraq Teachers			
	Frequency	Percent	Frequency	Percent		
Bachelor's degree	7	50/00	8	50/0		
Master's degree	7	50/00	7	43/80		
Ph.D. degree	-	-	1	6/30		
Total	14	100	16	100		

As it shows in Table (2), 50 percent of Iranian and Iraq teachers have Bachelors' degree, on other hand, the frequency of Iranian and Iraq teachers were equal in regard to Master's degree.

Table 3. Frequency of years in teaching mathematics

	Iranian T	eachers	Iraq Teachers			
	Frequency	Percent	Frequency	Percent		
Less than 5 years	3	21/40	2	12/50		
6-10 years	3	21/40	5	31/30		
11-15 years	5	35/70	5	31/30		
Over 15 years	3	21/40	4	25/00		
Total	14	100	16	100		

In Table (3), 11 to 15 years of teaching mathematics among Iranian and Iraq teachers have the most frequency.

Table 4. Frequency of current teaching grade

	Iranian T	eachers	Iraq Teachers			
	Frequency	Percent	Frequency	Percent		
Grade 8	3	21/40	2	12/50		
Grade 9	1	7/10	2	12/50		
Grade 10	3	21/40	5	31/30		
Grade 11	3	21/40	6	37/50		
Grade 12	4	28/60	1	6/30		
Total	14	100/0	16	100/0		

The results of Table (4) indicated that Iranian and Iraq teachers are teaching in grade 11 and grade 12 respectively.

Table 5. Frequency of age and mean of views

	Ira	nian Teachers	Iraq Teachers			
	Age	Mean Questions	Age	Mean Questions		
Mean	36/40 4/08		31/50	4/00		
Median	36	3/95	31	4/02		
Mode	33	3/78	Less than 30	4/00		
Std. Deviation	0/63	0/38	0/89	0/47		

Frequencies of Table (5) show the mean of Iranian and Iraq teachers' age and their views regard to all questions. As it clear, the mean of age is 36/40 and 31/50 for Iranian and Iraq teachers respectively and both two have response to "agree scale".

TABLE 6. Descriptive statistics of factors secondary Iranian and Iraq teachers of mathematics

		Iranian Teachers								
	TM	PPLP	RL	TI	HAA	CD	PTS	TMSR		
Mean	4/24	3/80	3/65	3/65	3/76	4/34	3/85	4/12		
Median	4/42	3/81	3/50	3/50	3/50	4/25	4/00	4/16		
Mode	4/43	3/50	3/00	3/00	3/50	4/25	4/00	4/00		
Std. Deviation	0/52	0/64	0/84	0/84	0/55	0/47	0/82	0/77		
				Iraq T	eacher:	s				
	TM	PPLP	RL	TI	HAA	CD	PTS	TMSR		
Mean	4/19	4/04	4/23	4/30	3/89	4/21	3/57	4/09		
Median	4/14	4/00	4/25	4/33	3/87	4/25	3/50	4/00		
Mode	3/71	3/63	4/50	4/33	3/00	4/25	3/00	3/67		
Std. Deviation	0/55	0/47	0/39	0/53	0/68	0/32	0/76	0/54		

The results of descriptive statistics of questionnaire's factors are determined in Table (6) accord to the views of Iranian and Iraq teachers of mathematics at secondary schools. At most Iranian teachers have response to agree scale for class discourse (CD) factor than other factors. Iraq teachers have selected agree scale for teacher inquiry (TI) factor than other factors mostly.

For testing exactly, researchers have used of One-Sample Kolmogorov-Smirnov Test and (K-S) and One-Sample Test in SPSS and then for testing views of teachers regard to each question, one sample sign test of MINITAB is used. Before testing by one sample test, data are studied by K-S test in order to studying normality. The results of K-S indicated that all data are normal (p > 0/05). Then, one sample t-test is run accord to Iranian and Iraq teachers of mathematics.

TABLE 7. The results of One-Sample Test of secondary Iranian and Iraq teachers of mathematics

	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Intervolution of the Difference	
					Lower	Upper
Mean for Iranian teachers	0/80	13	0/43	0/083	-0/14	0/30
Mean for Iraq teachers	0/00	15	0/98	0/00	-0/25	0/25

The results of Table (7) shows that null hypothesis is accepted (p > 0/05). In fact, the results of one sample t-test indicate that Iranian and Iraq teachers of mathematics have positive views regard to factors of questionnaire. Really, since they have positive views, it can conclude that both nations (accord to these samples) have applied problem solving procedure in math classes. Then each question of questionnaire is tested by MINITAB.

Table 8. The results of One Sample Sign of secondary Iranian and Iraq teachers of mathematics

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$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Q22	3	6	7	0/94	4	Q22	1	5	8	0/99	5
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Q23	3	7	6	0/91	4	Q23	2	9	3	0/81	4
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Q25	7	3	6	0/50	4	Q25	4	5	5	0/74	4
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		5	9	2	0/22	4	Q26	4	4	6	0/82	4
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		1	8	7	0/99	4	Q27	1	6	7	0/99	4/50
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	Q36	2	9	5	0/93	4	Q36	2	8	4	0/89	4

The results of one sample sign test indicated that Iranian and Iraq teachers of mathematics have selected "agree scale" for almost questions in Table (8). They believed that "teaching method (TM)", "Planning and Preparing the Lesson Plan (PPLP)", "Relationship with the Learner (RL)", "teacher inquiry (TI)", "homework assignments and activities (HAA)", "class discourse (CD)", "post-teaching sessions (PTS)", and "teaching materials (for studying and reviewing) (TMSR)" implement in problem solving approach of Iranian and Iraq mathematical teachers for math class at present.

11. Conclusion

Secondary school mathematics mostly concentrates on enhancing the students' understanding of, and skill in employing, symbolic concepts to describe and reason about variables, functions, inequalities, expressions and equations. In spite of spending a lot of time on teaching and practicing in class, students often do not successfully learn the basic school algorithms or unable to correctly use them in mathematical situations. Most of the students consider mathematics as a disconnected and meaningless process which is made of a set of rules for operations with symbols which do not signify anything useful or real. This negative perspective towards mathematics might be due to the teaching they had experienced in their secondary school mathematics classes. Therefore, to change such experience, a problem-solving approach was introduced to mathematics teaching and learning domain. Problem solving stands for the process of using mathematical skills and knowledge in unfamiliar and new situations. The aim of encouraging students to do problem solving tasks is to make them learn and able to use the mathematical thinking process, in a way that they can be able to use these processes whenever required. Three main areas of problem-solving issue, including teaching through problem solving, teaching about problem solving, and teaching for problem solving considered usually. Teaching for problem solving refers to the situation where the procedures are taught first by the teacher, and then the problems related to what was just taught are solved. Teaching about problem solving, refers to when teacher has taught different techniques such as drawing a graph or table to the students to use them as options for working on the problem; Teaching through problem solving refers to the use of problems for teaching the important mathematical concepts. In this area of problem solving, the problems are solved by the students in their own ways without the interference of the teacher, they can learn the main mathematical concepts and to do that, the students use their available mental tools; in fact, it needs more than just giving a correct form of problems to the students and asking them to solve those problems. Considering mathematics instruction, it is assumed that drill and practice methods are still used by the teachers, while problem-solving approach is an efficient alternative to such methods. It must be asserted that employing a problem-solving approach in class cannot take place overnight, and it must be considered as a long-term

investment. Gradual approach might be needed to prove teachers that the traditional methods they are using currently are no more effective and relevant to the requirements of the modern societies. To prove this to most of the teachers who hold such a viewpoint, it is recommended to create opportunities for them to successfully experience the problem-solving method; moreover, they must be asked to challenge their own teaching methods and critically reflect on them. Since Iranian teachers of mathematics have modified their strategies in recent years, we have studied the procedure of mathematical problem solving of Iranian math class versus Iraq math class. Singapore have many improvements in the field of mathematics education for many years. Iraq have followed main and effective procedures that Iraq students have the best performance in mathematics for many years in TIMSS. In contrast, Iranian students have weak performance in TIMSS for many years. It seems that teachers of primary level do not follow the procedures such as effective problem solving at first. At present, education systems of Iran have effort to improve the system of teaching mathematics through implementing in-service course for teachers. Then we have studied the views of Iranian teachers versus Singaporean teachers in order to determine the differences between two systems Mullis et al [15]. Through electronic questionnaire, many of data (in the form of case study-from capitals) are collected from both countries. As it shown from statistical tables, the view and procedures of both teachers of two countries were same. It seems that Iranian teachers of mathematics have used of elements of problem solving procedure at secondary level and since they did not apply effective problem-solving procedure at primary school, Iranian students have not considerable performance in TIMSS. The results of each factors of questionnaire indicated that Iranian teachers have response to agree scale for class discourse (CD) factor than other factors. Iraq teachers have selected agree scale for teacher inquiry (TI) factor than other factors mostly. Teacher inquiry is so complex process in math class that needs to work from elementary to secondary level. Iraq teachers have applied the inquiry at primary level, that, this case did not implement at primary school in Iran. In recent years, problem solving process is remarkable in all levels of Education in Iran. Then almost secondary students have not seen effective problem-solving process at first in Iran. Therefore, problem solving approach is innovative approach for secondary teachers and their students. In recent years, Iranian teachers have tried to change their styles of teaching in mathematics, geometry and algebra. Certainly, they have optimistic dreams about teaching base on problem solving approach. Problem solving is a modern approach that need to practice in Iran while Iraq teachers have applied problem solving approach at primary level to college level. Therefore, researcher have recommendation for future research regard to these findings: factors of this questionnaire can study among more teachers from Iraq and Iran, and it can study all factors among Iranian and Iraq teachers at all academic levels, the results of this research can modify the status of math class in both countries if volume of samples be more and more. Regard to problem solving frameworks,

researchers can apply stages for implementing the problem-solving approach that proposed by us for math class then the results of these strategies can study through interview with Iranian and Iraq teachers of mathematics at secondary level.

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References

- N. M. Aljaberi, & E. Gheith, Pre-Service Class Teacher'Ability in Solving Mathematical Problems and Skills in Solving Daily Problems, Higher Education Studies, 6., 3. (2016), 32-47
- [2] A. F. Artzt, E. Armour-Thomas, F. R. Curcio, & T. J. Gurl, Becoming a reflective mathematics teacher: A guide for observations and self-assessment. Routledge, (2015).
- [3] M. Barber, & M. Mourshed, How the world's best-performing schools systems come out on top, McKinsey & Company, (2007).
- [4] G. Guswinda, P. Yuanita, & N. M. Hutapea, Improvement of Mathematical Problem Solving and Disposition Ability of MTs Students through Strategies Think Talk Write in Cooperative Learning in Kuantan Singingi Regency, Journal of Educational Sciences, 3., 3. (2019), 377-389.
- [5] S. Heleni, & Z. Zulkarnain, The Influence of Mathematical Thinking Ability with Modified MOORE Method on Learning Outcomes of Basic Mathematic II Chemical Education Students, Journal of Educational Sciences, 2., 2. (2018), 33-41.
- [6] H. C. Hill, B. Rowan, & D. L. Ball, Effects of teachers' mathematical knowledge for teaching on student achievement, American educational research journal, 42., 2. (2005), 371-406.
- [7] H. C. Hill, D. L. Ball, & S. G. Schilling, Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students, Journal for research in mathematics education, 39., 4. (2008), 372-400.
- [8] J. Kilpatrick, J. Swafford, & B. Findell, Adding it up: Helping children learn mathematics, Washington, DC: National Academy Press, (Eds.), (2001).
- [9] B. Kaur, Mathematics education in Singapore-an insider's perspective, Journal on Mathematics Education, 5., 1. (2014), 1-16.
- [10] L. C. Larson, Problem-solving through problems, Springer Science & Business Media, (2012).
- [11] R. A. Lesh, & H. Doerr, MBeyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching, Routledge, (2003).
- [12] D. Loewenberg Ball, M. H. Thames, & G. Phelps, Content knowledge for teaching: What makes it special?, Journal of teacher education, 59., 5. (2008), 389-407.
- [13] S. J. Matlala, The experiences of secondary mathematics teachers teaching mathematics through problem solving (Doctoral dissertation, Stellenbosch: Stellenbosch University), (2015).
- [14] H. Mo'meni Mahmouei, A. Zangoee, M. R. Dehghani, The effect of Polya problem solving strategies on self-concept and mathematicas academic achievement of fifth graders, Research in curriculum, 11: (2015), 46-57.
- [15] I. V. Mullis, M. O. Martin, B. Fishbein, P. Foy, & S. Moncaleano, Findings from the TIMSS 2019 problem solving and inquiry tasks, (2021).
- [16] B. Nolan, M. Dempsey, J. Lovatt, & A. O'Shea, Developing mathematical knowledge for teaching (MKT) for pre-service teachers: A study of students' developing thinking in relation to the teaching of mathematics, Proceedings of the British Society for Research into Learning Mathematics, 35., 1. (2015).

- [17] A. Panaoura, Improving problem solving ability in mathematics by using a mathematical model: A computerized approach, Computers in Human Behavior, 28., 6. (2012), 2291-2297.
- [18] H. P. Phan, B. H. Ngu, & A. S. Yeung, Achieving optimal best: Instructional efficiency and the use of cognitive load theory in mathematical problem solving, Educational Psychology Review, 29., 4. (2017), 667-692.
- [19] G. Polya, How to solve it: A new aspect of mathematical method (Vol. 85), Princeton university press, (2004).
- [20] V. A. Reston, NCTM: National Council of Teachers of Mathematics, Principles and Standards for School Mathematics. Reston, VA: NCTM, (2000).
- [21] A. H. Schoenfeld, Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint), Journal of education, 196., 2. (2016), 1-38.
- [22] A. H. Schoenfeld, Cognitive science and mathematics education, Routledge, (2013).
- [23] M. A. Shouk, Mathematics Education in the Arab States, The Mathematics Teacher, 63., 4. (1970), 321-325.
- [24] L. S. Shulman, Those who understand: Knowledge growth in teaching, Educational researcher, 15., 2. (1986) 4-14.
- [25] L. Shulman, Knowledge and teaching: Foundations of the new reform, Harvard educational review, 57., 1. (1987), 1-23.
- [26] K. Stacey, The place of problem solving in contemporary mathematics curriculum documents, The Journal of Mathematical Behavior, 24., (3-4). (2005), 341-350.
- [27] Z. Veghar, A. Khaleghkhah, & A. Rezaee-Sharif, Study of efficiency of Polya problem solving instruction on mathematics self-efficiency of sixth garders in Ardebil, Third international conference of psychology, educational science and life style, (2017).
- [28] C. Xenofontos, & P. Andrews, Defining mathematical problems and problem solving: Prospective primary teachers' beliefs in Cyprus and England, Mathematics Education Research Journal, 26., 2. (2014), 279-299.

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