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LANGUAGES OF SINGLE-VALUED NEUTROSOPHIC GENERAL AUTOMATA

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Abstract. In this paper, we define the concepts of single-valued neutrosophic general automaton, complete and deterministic single-valued neutrosophic general automaton. We present a minimal single-valued neutrosophic general automaton that preserves the language for a given single-valued neutrosophic general automaton. Moreover, we present the closure properties such as union and intersection for single-valued neutrosophic general automata.

Keywords: Neutrosophic set, Automata, Intuitionistic set, Submachine, General fuzzy automata.
2020 MSC: 20M35, 68Q45, 68Q70

1. Introduction

The concept of 'fuzzy'together with a number of other notions in mathematics and other areas were fuzzified by Zadeh [36] in 1965. The applications of fuzzy sets have been found very useful in the domain of mathematics and elsewhere. In this real, among the early surveys was the concept of fuzzy automaton suggested by Wee [35] and Santos [22]. Doostfatemeh and Kremer [9] introduced the concept of general fuzzy automata.

Neutrosophy is one of the useful tools to deal with uncertainty in real-world issues. Neutrosophy is a branch of philosophy that was introduced by Florentin Smarandache [23–25]. A neutrosophic set is a general framework that generalizes the concept of fuzzy set, interval-valued fuzzy set, and intuitionistic fuzzy set. Wang et al. [34] introduced single-valued neutrosophic sets which is a neutrosophic set defined in the range [0, 1]. The neutrosophic set is an appropriate mechanism for interpreting real-life philosophical problems but not for scientific problems since it is difficult to consolidate. In neutrosophic sets, the degree of indeterminacy can be defined independently since it is quantified explicitly which led to different from intuitionistic fuzzy sets. A number of authors have been applied the concept of the neutrosophic set to many other structures especially in algebra [10, 16], decision-making [1, 2, 7, 17], medical [3, 4, 6], water

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quality management [20] and traffic control management [18, 19]. For more see [12–14].

Afterward, the concept of intuitionistic general fuzzy automaton was introduced and studied by Shamsizadeh and Zahedi [30]. For further information see the recent literature as [5,8,21,28,29,31,32]. The concept of interval neutrosophic finite state machine was introduced by Tahir Mahmood [15]. In 2019 [11] Kavikumar introduced the notion of neutrosophic general fuzzy automata.

In this note, by considering the notion of neutrosophic set, we extend the notion of the single-valued neutrosophic general machine and introduce the notion of single-valued neutrosophic general automaton and present a minimal single-valued neutrosophic general automaton. We give the concepts of complete and deterministic for single-valued neutrosophic general automaton.

2. Preliminaries

In this section, some concepts and definitions related to single-valued neutrosophy and automaton are introduced.

Definition 2.1. [9] A general fuzzy automaton (GFA) is considered as:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where (i) Q is a finite set of states, $Q = \{q_1, q_2, \dots, q_n\}$, (ii) Σ is a finite set of input symbols, $\Sigma = \{a_1, a_2, \dots, a_m\}$, (iii) $\tilde{R}$ is the set of fuzzy start states, $\tilde{R} \subseteq \tilde{P}(Q)$, (iv) Z is a finite set of output symbols, $Z = \{b_1, b_2, \dots, b_k\}$, (v) $\omega : Q \to Z$ is the output function, (vi) $\tilde{\delta} : (Q \times [0,1]) \times \Sigma \times Q \to [0,1]$ is the augmented transition function, (vii) function $F_1 : [0,1] \times [0,1] \to [0,1]$ is called membership assignment function. Function $F_1(\mu, \delta)$, as is seen, is motivated by two parameters μ and δ, where μ is the membership value of a predecessor and δ is the weight of a transition.

With this definition, the process that occurs upon the transition from state q_i to q_j an input a_k is characterized by:

$$\mu^{t+1}(q_j) = \tilde{\delta}((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \delta(q_i, a_k, q_j)).$$

It means that membership value (mv) of the state q_j at time $t+1$ is calculated by function F_1 utilizing both the membership value of q_i at time t and the weight of the transition.

There have been many options for the function $F_1(\mu, \delta)$. For instance, it can be $\max\{\mu, \delta\}$, $\min\{\mu, \delta\}$, $\dfrac{\mu + \delta}{2}$, or any other pertinent mathematical functions.

(viii) $F_2 : [0,1]^* \to [0,1]$, is called multi-membership resolution function. The multi-membership resolution function determines the multi-membership active states and allocates a single membership value to them.

We let $Q_{act}(t_i)$ be the set of all active state at time t_i, $\forall_i \geq 0$. We have $Q_{act}(t_0) = \tilde{R}$ and $Q_{act}(t_i) = \{(q, \mu^{t_i}(q)) | \exists q' \in Q_{act}(t_{i-1}), \exists a \in \Sigma, \delta(q', a, q) \in \Delta\}$, $\forall i \geq 1$. Since $Q_{act}(t_i)$ is a fuzzy set, to demonstrate that a state q belongs to $Q_{act}(t_i)$ and T is a subset of $Q_{act}(t_i)$, we should write: $q \in Domain(Q_{act}(t_i))$ and $T \subseteq Domain(Q_{act}(t_i))$; henceforth, we simply specify them by: $q \in Q_{act}(t_i)$ and $T \subseteq Q_{act}(t_i)$.

Definition 2.2. [26] Let Σ be a space of points (objects), with a generic element in Σ denoted by x. A neutrosophic set A in Σ is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A : \Sigma \to]0^-, 1^+[, I_A : \Sigma \to]0^-, 1^+[, F_A : \Sigma \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.3. [33] Single-valued neutrosophic set is the immediate results of neutrosophic set if it is defined over standard unit interval $[0, 1]$ instead of the non-standard unit interval $]0^-, 1^+[$. A single-valued neutrosophic subset (SVNS) A of Q is defined by $SVNS(A) = \{(x, T_A(x), I_A(x), F_A(x)) \big| x \in \Sigma\}$, where $T_A(x), I_A(x), F_A(x) : \Sigma \to [0, 1]$ such that $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2.4. [27] A single-valued neutrosophic general machine (SVNGM) $\mathcal{M}$ is a six-tuple machine denoted by $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$, where

1. Q is a finite set of states,
2. Σ is a finite set of input symbols,
3. $\tilde{R} \subseteq \tilde{P}(Q)$ is the set of single-valued neutrosophic initial states,
4. $\tilde{\delta} : (Q \times [0, 1] \times [0, 1] \times [0, 1]) \times \Sigma \times Q \to [0, 1] \times [0, 1] \times [0, 1]$ is the single-valued neutrosophic augmented transition function,
5. $E_1 = (E_1^T, E_1^I, E_1^F)$, where $E_1^T : [0, 1] \times [0, 1] \to [0, 1]$ is a t-norm and it called the truth-membership assignment function. $E_1^T(T, T_\delta)$ is motivated by two parameters T and T_δ, where T is the truth-membership value of a predecessor and T_δ is the truth-membership value of the transition. Also, $E_1^I : [0, 1] \times [0, 1] \to [0, 1]$ is a t-norm and it is called the indeterminacy-membership function. $E_1^I(I, I_\delta)$ is motivated by two parameters I and I_δ, where I is the indeterminacy-membership value of a predecessor and I_δ is the indeterminacy-membership value of the transition. Moreover, $E_1^F : [0, 1] \times [0, 1] \to [0, 1]$ is a t-conorm and it is called the falsity-membership function. $E_1^F(F, F_\delta)$ is motivated by two parameters F and F_δ, where F is the falsity-membership value of a predecessor and F_δ is the falsity-membership value of the transition. In this definition, the process that takes place upon the transition from the state q_i to q_j on an input a_k is represented by:

$$T^{t+1}(q_j) = \tilde{\delta}_1((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_j)),$$
$$I^{t+1}(q_j) = \tilde{\delta}_2((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_j)),$$
$$F^{t+1}(q_j) = \tilde{\delta}_3((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^F(T^t(q_i), \delta_3(q_i, a_k, q_j)),$$

where

$$\tilde{\delta}((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = (\tilde{\delta}_1((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j),$$
$$\tilde{\delta}_2((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j), \tilde{\delta}_3((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j)),$$

and

$$\delta(q_i, a_k, q_j) = (\delta_1(q_i, a_k, q_j), \delta_2(q_i, a_k, q_j), \delta_3(q_i, a_k, q_j)).$$

6. $E_2 = (E_2^T, E_2^I, E_2^F)$, where $E_2^T : [0,1] \times [0,1] \to [0,1]$ is a T-conorm and it is called multi-truth-membership function, $E_2^I : [0,1] \times [0,1] \to [0,1]$ is a T-conorm and it is called multi-indetermincy-membership function, $E_2^F : [0,1] \times [0,1] \to [0,1]$ is a T-norm and it is called multi-falsity-membership function.

3. Languages of single-valued neutrosophic general automata

Definition 3.1. A single-valued neutrosophic general automaton (SVNGA) $\mathcal{M}$ is a eight-tuple machine denoted by $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$, where $(Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, \tilde{\delta}, E_1, E_2)$ is a SVNGM and

- Z is a finite non-fuzzy set of output symbols,
- $\tilde{\omega} : (Q \times [0,1] \times [0,1] \times [0,1]) \times Z \to [0,1] \times [0,1] \times [0,1]$ is the single-valued neutrosophic output function, where

$$\omega(q, b) = (\omega_1(q, b), \omega_2(q, b), \omega_3(q, b)),$$

and we define

$$\tilde{\omega}^t((q, T^t(q), I^t(q), F^t(q)), b) = (\tilde{\omega}_1^t((q, T^t(q), I^t(q), F^t(q)), b),$$
$$\tilde{\omega}_2^t((q, T^t(q), I^t(q), F^t(q)), b), \tilde{\omega}_3^t((q, T^t(q), I^t(q), F^t(q)), b)),$$

such that

$$\tilde{\omega}_1^t((q, T^t(q), I^t(q), F^t(q)), b) = T^t(q) \wedge \omega_1(q, b),$$
$$\tilde{\omega}_2^t((q, T^t(q), I^t(q), F^t(q)), b) = I^t(q) \wedge \omega_2(q, b),$$
$$\tilde{\omega}_3^t((q, T^t(q), I^t(q), F^t(q)), b) = F^t(q) \vee \omega_3(q, b).$$

Example 3.2. *Let SVNGA* $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ *such that* $Q = \{p, q, r\}, \Sigma = \{a\}$, $\tilde{R} = \{(p, 0.3, 0.4, 0.9)\}, Z = \{b\}$, δ *and* ω *are defined as follows:*

$$\delta(p, a, p) = (0.3, 0.3, 0.3), \qquad \delta(p, a, q) = (0.4, 0.7, 1),$$
$$\delta(p, a, r) = (0.5, 0.5, 0.6), \qquad \delta(q, a, p) = (0.6, 0.7, 1),$$
$$\delta(q, a, r) = (0.4, 0.6, 0.2), \qquad \delta(r, a, q) = (0.1, 0.7, 0.8),$$

$$\omega(p, b) = (0.4, 0.2, 0.7), \qquad \omega(q, b) = (0.4, 0.2, 0.9), \qquad \omega(r, b) = (0, 0, 1).$$

Now, we can consider E_1 as follows:
1. $E_1^T = T \wedge T_\delta,\ E_1^I = I \wedge I_\delta,\ E_1^F = F \vee F_\delta,$

$$T^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_m)),$$
$$I^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m) = \bigwedge_{i=1}^{n} E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m)),$$

2. $E_1^T = T.T_\delta,\ E_1^I = I.I_\delta,\ E_1^F = F + F_\delta - F.F_\delta,$

$$T^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_m)),$$
$$I^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m) = \bigwedge_{i=1}^{n} E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m)),$$

3. $E_1^T = T \wedge T_\delta,\ E_1^I = I \wedge I_\delta,\ E_1^F = F \vee F_\delta,$

$$T^{t+1}(q_m) = T_p(T_p(T^t(q_i), \delta_1(q_i, a_k, q_m))),$$
$$I^{t+1}(q_m) = T_p(T_p(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m)) = S_p(S_p E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m))),$$

where T_p is the product t-norm and S_p is the product t-conorm.
If we choose the case 1, then we have

$$
\begin{aligned}
T^{t_1}(p) &= E_1^T(T^{t_0}(p), \delta_1(p,a,p)) = 0.3 \wedge 0.3 = 0.3,\\
I^{t_1}(p) &= E_1^I(I^{t_0}(p), \delta_2(p,a,p)) = 0.4 \wedge 0.3 = 0.3,\\
F^{t_1}(p) &= E_1^F(F^{t_0}(p), \delta_3(p,a,p)) = 0.9 \vee 0.3 = 0.9,\\
T^{t_1}(q) &= E_1^T(T^{t_0}(p), \delta_1(p,a,q)) = 0.3 \wedge 0.4 = 0.3,\\
I^{t_1}(q) &= E_1^I(I^{t_0}(p), \delta_2(p,a,q)) = 0.4 \wedge 0.7 = 0.4,\\
F^{t_1}(q) &= E_1^F(F^{t_0}(p), \delta_3(p,a,q)) = 0.9 \vee 1 = 1,\\
T^{t_1}(r) &= E_1^T(T^{t_0}(p), \delta_1(p,a,r)) = 0.3 \wedge 0.5 = 0.3,\\
I^{t_1}(r) &= E_1^I(I^{t_0}(p), \delta_2(p,a,r)) = 0.4 \wedge 0.5 = 0.4,\\
F^{t_1}(r) &= E_1^F(F^{t_0}(p), \delta_3(p,a,r)) = 0.9 \vee 0.6 = 0.9,\\
T^{t_2}(p) &= E_1^T(T^{t_1}(p), \delta_1(p,a,p)) \vee E_1^T(T^{t_1}(q), \delta_1(q,a,p)) = (0.3 \wedge 0.3) \vee (0.3 \wedge 0.6) = 0.3,\\
I^{t_2}(p) &= E_1^I(I^{t_1}(p), \delta_2(p,a,p)) \vee E_1^I(I^{t_1}(q), \delta_2(q,a,p)) = (0.3 \wedge 0.3) \vee (0.4 \wedge 0.7) = 0.4,\\
F^{t_2}(p) &= E_1^F(F^{t_1}(p), \delta_3(p,a,p)) \wedge E_1^F(F^{t_1}(q), \delta_3(q,a,p)) = (0.9 \vee 0.3) \wedge (1 \vee 1) = 0.9,\\
T^{t_2}(q) &= E_1^T(T^{t_1}(p), \delta_1(p,a,q)) \vee E_1^T(T^{t_1}(r), \delta_1(r,a,q)) = (0.3 \wedge 0.4) \vee (0.3 \wedge 0.1) = 0.3,\\
I^{t_2}(q) &= E_1^I(I^{t_1}(p), \delta_2(p,a,q)) \vee E_1^I(I^{t_1}(r), \delta_2(r,a,q)) = (0.3 \wedge 0.7) \vee (0.4 \wedge 0.7) = 0.4,\\
F^{t_2}(q) &= E_1^F(F^{t_1}(p), \delta_3(p,a,q)) \wedge E_1^F(F^{t_1}(r), \delta_3(r,a,q)) = (0.9 \vee 1) \wedge (0.9 \vee 0.8) = 0.9,\\
T^{t_2}(r) &= E_1^T(T^{t_1}(p), \delta_1(p,a,r)) \vee E_1^T(T^{t_1}(q), \delta_1(q,a,r)) = (0.3 \wedge 0.5) \vee (0.3 \wedge 0.4) = 0.3,\\
I^{t_2}(r) &= E_1^I(I^{t_1}(p), \delta_2(p,a,r)) \vee E_1^I(I^{t_1}(q), \delta_2(q,a,r)) = (0.3 \wedge 0.5) \vee (0.4 \wedge 0.6) = 0.4,\\
F^{t_2}(r) &= E_1^F(F^{t_1}(p), \delta_3(p,a,r)) \wedge E_1^F(F^{t_1}(q), \delta_3(q,a,r)) = (0.9 \vee 0.6) \wedge (1 \vee 0.2) = 0.4.
\end{aligned}
$$

also,

$$
\begin{aligned}
&\omega_1^{t_0}((p,T^{t_0}(p),I^{t_0}(p),F^{t_0}(p)),b) = 0.3 \wedge 0.4 = 0.3,\\
&\omega_2^{t_0}((p,T^{t_0}(p),I^{t_0}(p),F^{t_0}(p)),b) = 0.4 \wedge 0.2 = 0.2,\\
&\omega_3^{t_0}((p,T^{t_0}(p),I^{t_0}(p),F^{t_0}(p)),b) = 0.9 \vee 0.7 = 0.9,\\
&\omega_1^{t_1}((p,T^{t_1}(p),I^{t_1}(p),F^{t_1}(p)),b) = 0.3 \wedge 0.4 = 0.3,\\
&\omega_2^{t_1}((p,T^{t_1}(p),I^{t_1}(p),F^{t_1}(p)),b) = 0.3 \wedge 0.2 = 0.2,\\
&\omega_3^{t_1}((p,T^{t_1}(p),I^{t_1}(p),F^{t_1}(p)),b) = 0.9 \vee 0.7 = 0.9,\\
&\omega_1^{t_1}((q,T^{t_1}(q),I^{t_1}(p),F^{t_1}(q)),b) = 0.3 \wedge 0.4 = 0.3,\\
&\omega_2^{t_1}((q,T^{t_1}(q),I^{t_1}(p),F^{t_1}(q)),b) = 0.4 \wedge 0.2 = 0.2,\\
&\omega_3^{t_1}((q,T^{t_1}(q),I^{t_1}(p),F^{t_1}(q)),b) = 1 \vee 0.9 = 1,\\
&\omega_1^{t_1}((r,T^{t_1}(r),I^{t_1}(r),F^{t_1}(r)),b) = 0.3 \wedge 0 = 0,\\
&\omega_2^{t_1}((r,T^{t_1}(r),I^{t_1}(r),F^{t_1}(r)),b) = 0.4 \wedge 0 = 0,\\
&\omega_3^{t_1}((r,T^{t_1}(r),I^{t_1}(r),F^{t_1}(r)),b) = 0.9 \vee 1 = 1.
\end{aligned}
$$

Clearly, we can see that there are some simultaneous transition to the action states p and q at time t_2.

Definition 3.3. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA. We define max-min SVNGA $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}^*, E_1, E_2)$ such that $\tilde{\delta}^* : Q_{act} \times \Sigma^* \times Q \to [0,1] \times [0,1] \times [0,1]$, where $Q_{act} = \{Q_{act}(t_0), Q_{act}(t_1), ...\}$ and for every $i \geq 0$,

$$\tilde{\delta}_1^*((q,T^t(q),I^t(q),F^t(q)),\Lambda,p) = \begin{cases} 1 & \text{if p=q} \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

$$\tilde{\delta}_2^*((q,T^t(q),I^t(q),F^t(q)),\Lambda,p) = \begin{cases} 1 & \text{if p=q} \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

$$\tilde{\delta}_3^*((q,T^t(q),I^t(q),F^t(q)),\Lambda,p) = \begin{cases} 0 & \text{if p=q} \\ 1 & \text{otherwise} \end{cases}, \tag{3}$$

and for every $i \geq 1$, $\tilde{\delta}^*((q,T^t(q),I^t(q),F^t(q)),a,p) = \tilde{\delta}((q,T^t(q),I^t(q),F^t(q)),a,p)$ and recursively,

$$
\begin{aligned}
\tilde{\delta}_1^*((q,T^t(q),I^t(q),F^t(q)),a_1a_2...a_n,p) = \vee\{&\tilde{\delta}_1((q,T^t(q),I^t(q),F^t(q)),a_1,p_1) \wedge \cdots \\
&\wedge \tilde{\delta}_1((p_{n-1},T^t(p_{n-1}),I^t(p_{n-1}),F^t(p_{n-1})),a_n,p) \\
&\big| p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), ..., p_{n-1} \in Q_{act}(t_{n-1})\},
\end{aligned}
$$

$$\tilde{\delta}_2^*((q,T^t(q),I^t(q),F^t(q)),a_1a_2...a_n,p) = \vee\{\tilde{\delta}_2((q,T^t(q),I^t(q),F^t(q)),a_1,p_1)\wedge\cdots \wedge\tilde{\delta}_2((p_{n-1},T^t(p_{n-1}),I^t(p_{n-1}),F^t(p_{n-1})),a_n,p) \big| p_1\in Q_{act}(t_1),p_2\in Q_{act}(t_2),...,p_{n-1}\in Q_{act}(t_{n-1})\},$$

$$\tilde{\delta}_3^*((q,T^t(q),I^t(q),F^t(q)),a_1a_2...a_n,p) = \wedge\{\tilde{\delta}_3((q,T^t(q),I^t(q),F^t(q)),a_1,p_1)\vee\cdots \vee\tilde{\delta}_3((p_{n-1},T^t(p_{n-1}),I^t(p_{n-1}),F^t(p_{n-1})),a_n,p) \big| p_1\in Q_{act}(t_1),p_2\in Q_{act}(t_2),...,p_{n-1}\in Q_{act}(t_{n-1})\},$$

in which $a_i\in\Sigma$, for all $1\le i\le n$ and assuming that the entered input at time t_i is a_i for $1\le i\le n-1$.

Example 3.4. *Let SVNGA $\mathcal{M}=(Q,\Sigma,\tilde{R}=\{(q_0,1,1,0)\},Z,\tilde{\omega},\tilde{\delta},E_1,E_2)$ in Example 3.2. By choosing case 1 in Example 3.2, we obtain a max-min SVNGA.*

In the rest of paper, instead of max-min SVNGA we say that SVNGA.

Definition 3.5. Let $\mathcal{M}=(Q,\Sigma,\tilde{R}=\{(q_0,1,1,0)\},Z,\tilde{\omega},\tilde{\delta},E_1,E_2)$ be a SVNGA. Define the relation R on Σ^* as follows: xRy if and only if there exists $p\in Q$ such that

$$\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0,$$
$$\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0,$$
$$\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)<1\Leftrightarrow\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)<1.$$

Theorem 3.6. *Let $\mathcal{M}=(Q,\Sigma,\tilde{R}=\{(q_0,1,1,0)\},Z,\tilde{\omega},\tilde{\delta},E_1,E_2)$ be a SVNGA. The relation R defined in Definition 3.5, is an equivalence relation on Σ^*.*

Proof. Clearly, xRx and if xRy, then yRx. Now, let xRy and yRx. Then

$$\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0,$$
$$\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0,$$
$$\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)<1\Leftrightarrow\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)<1,$$

also,

$$\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0\Leftrightarrow\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)>0,$$
$$\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)>0\Leftrightarrow\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)>0,$$
$$\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),y,p)<1\Leftrightarrow\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)<1.$$

Therefore, we have

$$\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_1^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)>0,$$
$$\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)>0\Leftrightarrow\tilde{\delta}_2^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)>0,$$
$$\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,p)<1\Leftrightarrow\tilde{\delta}_3^*((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),z,p)<1.$$

Therefore, R is an equivalence relation on Σ^*. □

Definition 3.7. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA. $\mathcal{M}$ is called a complete SVNGA if for every $p \in Q$ and $a \in \Sigma$, there exists at least one state $q \in Q$ such that

$$\delta_1(p, a, q) > 0,$$
$$\delta_2(p, a, q) > 0,$$
$$\delta_3(p, a, q) < 1.$$

Example 3.8. *Let SVNGA $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ as in Example 3.13. $\mathcal{M}$ is a complete SVNGA.*

Definition 3.9. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA and $x \in \Sigma^*$. Then x is called to be recognized by $\mathcal{M}$ if

$$\tilde{\delta}_1^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_1(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_2^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_2(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_3^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \vee \tilde{\omega}_3(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) < 1,$$

for some $p \in Q$. Also, $L(\mathcal{M}) = \{x \in \Sigma^* | x \text{ is recognized by } \mathcal{M}\}$ is called the language recognized by $\mathcal{M}$.

A set $L \subseteq \Sigma^*$ is called a recognizable set if there exists a SVNGA $\mathcal{M}$ such that $L(\mathcal{M}) = L$

Example 3.10. *Let SVNGA $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ in Example 3.2. Then $L(\mathcal{M}) = a^*$.*

Theorem 3.11. *Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA such that $T^{t_0}(q_0) > 0, I^{t_0}(q_0) > 0$ and $F^{t_0}(q_0) < 1$. Then there exists a complete SVNGA $\mathcal{M}^c$ such that $L(\mathcal{M}^c) = L(\mathcal{M})$.*

Proof. Let $\mathcal{M}^c = (Q^c, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}^c, \tilde{\delta}^c, E_1, E_2)$ such that $Q^c = Q \cup \{t\}, t \notin Q$, and δ^c and ω^c be as follows: for every $p, q \in Q$ and $a \in \Sigma$, $\delta^c(p, a, q) = \delta(p, a, q)$. Consider $\delta^c(p, a, t) = (0.5, 0.5, 0.5)$ if for every $q \in Q$, $\delta(p, a, q) = (c_1, c_2, c_3)$, where $c_1 = 0$ or $c_2 = 0$ or $c_3 = 1$. Also, let $\delta^c(t, a, t) = (0.5, 0.5, 0.5)$ and $\omega^c(p, b) = \omega(p, b)$, for every $p \in Q$ and $b \in Z$ and let $\omega^c(t, b) = (0, 0, 1)$. Clearly, $\mathcal{M}^c$ is a complete SVNGA and $L(\mathcal{M}) \subseteq L(\mathcal{M}^c)$.
Now, let $a_1 a_2 ... a_n = x \in L(\mathcal{M}^c)$. Then there exist $p, q \in Q^c$ such that

$$\tilde{\delta}_1^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_1^c(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_2^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_2^c(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_3^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \vee \tilde{\omega}_3^c(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) < 1.$$

Since $\tilde{\omega}_1^c(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0$, we have $T^{t_0+|x|}(p) > 0$ and $\omega_1^c(p, b) > 0$. So, $p \neq t$. Since $\tilde{\delta}_1^{*c}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0$, we have there exist $q_1, q_2, ..., q_{n-1} \in Q$ such that

$$\tilde{\delta}_1^c((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), a_1, q_1) > 0,$$
$$\tilde{\delta}_1^c((q_1, T^{t_1}(q_1), I^{t_1}(q_1), F^{t_1}(q_1)), a_2, q_2) > 0,$$
$$\vdots$$
$$\tilde{\delta}_1^c((q_{n-1}, T^{t_{n-1}}(q_{n-1}), I^{t_{n-1}}(q_{n-1}), F^{t_{n-1}}(q_{n-1})), a_n, p) > 0.$$

Then $T^{n-1}(q_{n-1}) > 0$ and $\delta_1(q_{n-1}, a_n, p) > 0$, so $q_{n-1} \in Q$. In a similar way, we can see that $q_1, q_2, ..., q_{n-1} \in Q$ and $\tilde{\delta}_1((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0$. Similarly, $\tilde{\delta}_2((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0$ and $\tilde{\delta}_3((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) < 1$. Therefore, $x \in L(\mathcal{M})$. Hence, $L(\mathcal{M}) = L(\mathcal{M}^c)$. □

Definition 3.12. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA. Then $\mathcal{M}$ is called a deterministic SVNGA if for every $q \in Q$ and $a \in \Sigma$ there exists at most a state $p \in Q$ such that $\delta(q, a, p) = (c_1, c_2, c_3)$, where $c_1 > 0, c_2 > 0$ and $c_3 < 1$.

Example 3.13. *Let SVNGA $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ in Example 3.2. $\mathcal{M}$ is not a deterministic SVNGA.*

Theorem 3.14. *Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a SVNGA. Then there exists a deterministic SVNGA $\mathcal{M}_d$ such that $L(\mathcal{M}) = L(\mathcal{M}_d)$.*

Proof. Let $\mathcal{M}_d = (Q_d, \Sigma, \tilde{R}_d, Z, \tilde{\omega}_d, \tilde{\delta}_d, E_1, E_2)$, where $Q_d = \{Q_x \big| x \in \Sigma^*\}$, such that

$$\begin{aligned} Q_x = \{p \in Q \big| & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0, \\ & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0, \\ & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) < 1\}, \end{aligned}$$

then

$$\begin{aligned} Q_\Lambda = \{p \in Q \big| & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), \Lambda, p) > 0, \\ & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), \Lambda, p) > 0, \\ & \tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), \Lambda, p) < 1\} \\ & = \{q_0\}. \end{aligned}$$

Consider $\tilde{R}_d = \{(Q_\Lambda, T^{t_0}(Q_\Lambda) = T^{t_0}(q_0), I^{t_0}(Q_\Lambda) = I^{t_0}(q_0), F^{t_0}(Q_\Lambda) = F^{t_0})\}$,

$$\delta_d(Q_x, a, Q_y) = \begin{cases} (0.5, 0.5, 0.5) & \text{if } Q_{xa} = Q_y \\ (0, 0, 1) & \text{otherwise} \end{cases}, \tag{4}$$

$\omega_d(Q_x, b) = (0.5, 0.5, 0.5)$ if $\omega(q, b) = (c_1, c_2, c_3)$, where $c_1 \neq 0 \neq c_2$, and $c_3 \neq 1$, for some $q \in Q_x$. Clearly, $\mathcal{M}_d$ is a deterministic SVNGA. Now, we prove that $L(\mathcal{M}_d) = L(\mathcal{M})$. Let $x \in L(\mathcal{M})$. Then there exists $p \in Q$ such that

$$\tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_1(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_2^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \wedge \tilde{\omega}_2(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) > 0,$$
$$\tilde{\delta}_3^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) \vee \tilde{\omega}_3(p, T^{t_0+|x|}(p), I^{t_0+|x|}(p), F^{t_0+|x|}(p)), b) < 1.$$

So, we have

$$\tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0,$$
$$\tilde{\delta}_2^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0,$$
$$\tilde{\delta}_3^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) < 1.$$

Then $p \in Q_x$. By definition δ_d and ω_d we have

$$\tilde{\delta}_{d1}^*((Q_\Lambda, T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_x) > 0,$$
$$\tilde{\delta}_{d2}^*((Q_\Lambda, T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_x) > 0,$$
$$\tilde{\delta}_{d3}^*((Q_\Lambda, T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_x) < 1,$$

also

$$\tilde{\omega}_{d1}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) > 0,$$
$$\tilde{\omega}_{d2}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) > 0,$$
$$\tilde{\omega}_{d3}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) < 1.$$

Therefor, $x \in L(\mathcal{M}_d)$ and $L(\mathcal{M}) \subseteq L(\mathcal{M}_d)$. Now, let $x \in L(\mathcal{M}_d)$. Then there exists $Q_y \in Q_d$ such that

$$\begin{aligned}\tilde{\delta}_{d1}^*((Q_\Lambda,&T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_y)\\ &\wedge \tilde{\omega}_{d1}(Q_y, T^{t_0+|x|}(Q_y), I^{t_0+|x|}(Q_y), F^{t_0+|x|}(Q_y)), b) > 0,\\ \tilde{\delta}_{d2}^*((Q_\Lambda,&T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_y)\\ &\wedge \tilde{\omega}_{d2}(Q_y, T^{t_0+|x|}(Q_y), I^{t_0+|x|}(Q_y), F^{t_0+|x|}(Q_y)), b) > 0,\\ \tilde{\delta}_{d3}^*((Q_\Lambda,&T^{t_0}(Q_\Lambda), I^{t_0}(Q_\Lambda), F^{t_0}(Q_\Lambda)), x, Q_y)\\ &\vee \tilde{\omega}_{d3}(Q_y, T^{t_0+|x|}(Q_y), I^{t_0+|x|}(Q_y), F^{t_0+|x|}(Q_y)), b) < 1.\end{aligned}$$

By definition δ_d, we have $Q_y = Q_x$. Since

$$\tilde{\omega}_{d1}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) > 0,$$
$$\tilde{\omega}_{d2}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) > 0,$$
$$\tilde{\omega}_{d3}(Q_x, T^{t_0+|x|}(Q_x), I^{t_0+|x|}(Q_x), F^{t_0+|x|}(Q_x)), b) < 1,$$

then $T^{t_0+|x|} > 0$, $\omega_{d1}(Q_x, b) > 0$ and $I^{t_0+|x|} > 0$, $\omega_{d2}(Q_x, b) > 0$ also, $F^{t_0+|x|} < 1$, $\omega_{d2}(Q_x, b) < 1$. So, there exists $p \in Q_x$ such that $\omega_1(p, b) > 0, \omega_2(p, b) > 0$ and $\omega_3(p, b) < 1$, also

$$\tilde{\delta}_1^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0,$$
$$\tilde{\delta}_2^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) > 0,$$
$$\tilde{\delta}_3^*((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, p) < 1.$$

Therefore, $x \in L(\mathcal{M})$. Hence, $L(\mathcal{M}_d) = L(\mathcal{M})$. □

Definition 3.15. Let $\mathcal{M}$ be a complete deterministic SVNGA. Then we say that $\mathcal{M}$ is minimal if for every complete deterministic SVNGA $\mathcal{M}'$ such that $L(\mathcal{M}) = L(\mathcal{M}')$, we have $|\mathcal{M}| \leq |\mathcal{M}'|$, where $|\mathcal{M}|$ is the number of states SVNGA $\mathcal{M}$.

Remark 3.16. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a complete deterministic SVNGA and R be the equivalence relation defined in Definition 3.5. Then the number of classes of equivalence relation R is not more than the number of states Q.

Definition 3.17. Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a complete deterministic SVNGA and R be the equivalence relation defined in Definition 3.5. Let Q' be the set of equivalence classes of R, i.e., $Q' = \{[x] \big| [x] \text{ is an equivalence class of } R\}$, $R' = \{([\Lambda], 1, 1, 0)\}$. Define $\delta' : Q' \times \Sigma \times Q' \to [0, 1] \times [0, 1] \times [0, 1]$ by

$$\delta'([x], a, [y]) = \begin{cases} (1, 1, 0) & \text{if } [xa] = [y] \\ (0, 0, 1) & \text{otherwise} \end{cases}, \tag{5}$$

and $\omega' : Q' \times Z \to [0, 1] \times [0, 1] \times [0, 1]$ by

$$\omega'([x], a) = \begin{cases} (1, 1, 0) & \text{if } x \in L(\mathcal{M}) \\ (0, 0, 1) & \text{otherwise} \end{cases}. \tag{6}$$

Clearly, $\mathcal{M}' = (Q', \Sigma, \tilde{R}' = \{([\Lambda], 1, 1, 0)\}, Z, \tilde{\omega}', \tilde{\delta}', E_1, E_2)$ is a complete deterministic SVNGA.

Theorem 3.18. *Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a complete deterministic SVNGA and $\mathcal{M}' = (Q', \Sigma, \tilde{R}' = \{([\Lambda], 1, 1, 0)\}, Z, \tilde{\omega}', \tilde{\delta}', E_1, E_2)$ be the SVNGA defined in Definition 3.17. Then $L(\mathcal{M}') = L(\mathcal{M})$.*

Proof. Let $x \in L(\mathcal{M})$. Then by definition δ' and ω' we have

$$\tilde{\delta}_1'^*(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [x]) = 1,$$
$$\tilde{\delta}_2'^*(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [x]) = 1,$$
$$\tilde{\delta}_3'^*(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [x]) = 0,$$

also,

$$\tilde{\omega}_1^{'}([x], T^{t_0+|x|}([x]), I^{t_0+|x|}([x]), F^{t_0+|x|}([x])), b) = 1,$$
$$\tilde{\omega}_2^{'}([x], T^{t_0+|x|}([x]), I^{t_0+|x|}([x]), F^{t_0+|x|}([x])), b) = 1,$$
$$\tilde{\omega}_3^{'}([x], T^{t_0+|x|}([x]), I^{t_0+|x|}([x]), F^{t_0+|x|}([x])), b) = 0.$$

Therefore, $x \in L(\mathcal{M}')$. Now, let $x \in L(\mathcal{M}')$. Then

$$\begin{aligned}
&\tilde{\delta}_1^{'*}(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [y]) \\
&\qquad \wedge \tilde{\omega}_1^{'}([y], T^{t_0+|x|}([y]), I^{t_0+|x|}([y]), F^{t_0+|x|}([y])), b) > 0, \\
&\tilde{\delta}_2^{'*}(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [y]) \\
&\qquad \wedge \tilde{\omega}_2^{'}([y], T^{t_0+|x|}([y]), I^{t_0+|x|}([y]), F^{t_0+|x|}([y])), b) > 0, \\
&\tilde{\delta}_3^{'*}(([\Lambda], T^{t_0}([\Lambda]), I^{t_0}([\Lambda]), F^{t_0}([\Lambda])), x, [y]) \\
&\qquad \vee \tilde{\omega}_3^{'}([y], T^{t_0+|x|}([y]), I^{t_0+|x|}([y]), F^{t_0+|x|}([y])), b) < 1.
\end{aligned}$$

By definition δ', we have $[y] = [x]$, and by definition ω', we have $\omega_1'([x], b) > 0$, $\omega_2'([x], b) > 0$ and $\omega_3'([x], b) < 1$. Then $x \in L(\mathcal{M})$. Then the claim holds. □

Theorem 3.19. *Let $\mathcal{M} = (Q, \Sigma, \tilde{R} = \{(q_0, 1, 1, 0)\}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ be a complete deterministic SVNGA. Then $\mathcal{M}'$ defined in Definition 3.17, is a minimal SVNGA such that $L(\mathcal{M}) = L(\mathcal{M}')$.*

Proof. By Remark 3.16 and Theorem 3.18, it is obvious.

□

4. Closure properties of single-valued neutrosophic general automata

Theorem 4.1. *Let L_1 and L_2 be two recognizable sets. Then $L_1 \cup L_2$ is a recognizable set too.*

Proof. Let $\mathcal{M}_1 = (Q_1, \Sigma, \tilde{R}_1, Z, \tilde{\omega}^1, \tilde{\delta}^1, E_1, E_2)$ and $\mathcal{M}_2 = (Q_2, \Sigma, \tilde{R}_2, Z, \tilde{\omega}^2, \tilde{\delta}^2, E_1, E_2)$ be two SVNGAs such that $L(\mathcal{M}_1) = L_1$ and $L(\mathcal{M}_2) = L_2$ and let $Q_1 \cap Q_2 = \emptyset$. Consider $\mathcal{M} = (Q, \Sigma, \tilde{R}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$, where $Q = Q_1 \cup Q_2$, $\tilde{R} = \tilde{R}_1 \cup \tilde{R}_2$, δ and ω are defined as follows:

$$\delta(q, a, p) = \begin{cases} \delta^1(q, a, p) & \text{if } q, p \in Q_1 \\ \delta^2(q, a, p) & \text{if } q, p \in Q_2 \\ (0, 0, 1) & \text{otherwise} \end{cases}, \tag{7}$$

$$\omega(q, b) = \begin{cases} \omega^1(q, b) & \text{if } q \in Q_1 \\ \omega^2(q, b) & \text{if } q \in Q_2 \end{cases}, \tag{8}$$

Now, we show that $L(\mathcal{M}) = L_1 \cup L_2$. Let $x = a_1a_2...a_n \in L(\mathcal{M})$. Then there exist $p, q \in Q$ and $b \in Z$ such that

$$\tilde{\delta}_1^*((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) \wedge \tilde{\omega}_1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\delta}_2^*((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) \wedge \tilde{\omega}_2((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\delta}_3^*((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) \vee \tilde{\omega}_2((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) < 1.$$

So, there are $p_1, p_2, ..., p_{n-1} \in Q$ such that

$$\tilde{\delta}_1((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), a_1, p_1) > 0,$$
$$\tilde{\delta}_1((p_1, T^{t_1}(p_1), I^{t_1}(p_1), F^{t_1}(p_1)), a_2, p_2) > 0,$$
$$\vdots$$
$$\tilde{\delta}_1((p_{n-1}, T^{t_{n-1}}(p_{n-1}), I^{t_{n-1}}(p_{n-1}), F^{t_{n-1}}(p_{n-1})), a_n, q) > 0.$$

Therefore, $T^{t_0}(p) > 0$, $\delta_1(p, a_1, p_1) > 0$, $\delta_1(p_1, a_2, p_2) > 0, ..., \delta_1(p_{n-1}, a_n, q) > 0$. Then by definition of δ, we have $p, p_1, ..., p_{n-1}, q \in Q_1$ or $p, p_1, p_2, ..., p_{n-1}, q \in Q_2$. Without loss of generality, let $p, p_1, p_2, ..., p_{n-1}, q \in Q_1$. Then $\tilde{\delta}_1^{*1}((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) > 0$. Similarly, $\tilde{\delta}_2^{*1}((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) > 0$, $\tilde{\delta}_3^{*1}((p, T^{t_0}(p), I^{t_0}(p), F^{t_0}(p)), x, q) < 1$. Also, by definition of ω, $\tilde{\omega}_1^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0$, $\tilde{\omega}_2^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0$ and $\tilde{\omega}_3^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) < 1$. So, $x \in L(\mathcal{M}_1)$ and $L(\mathcal{M}) \subseteq L(\mathcal{M}_1) \cup L(\mathcal{M}_2)$. By definition of $\mathcal{M}$, it is clear that $L(\mathcal{M}_1) \cup L(\mathcal{M}_2) \subseteq L(\mathcal{M})$. Hence, $L(\mathcal{M}_1) \cup L(\mathcal{M}_2) = L(\mathcal{M})$. □

Theorem 4.2. *Let L_1 and L_2 be two recognizable sets. Then $L_1 \cap L_2$ is a recognizable set too.*

Proof. Since L_1 and L_2 are recognizable sets, there exist two complete deterministic SVNGAs $\mathcal{M}_1 = (Q_1, \Sigma, \tilde{R}_1 = (q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), Z, \tilde{\omega}^1, \tilde{\delta}^1, E_1, E_2)$ and $\mathcal{M}_2 = (Q_2, \Sigma, \tilde{R}_2 = (p_0, T^{t_0}(p_0), I^{t_0}(p_0), F^{t_0}(p_0)), Z, \tilde{\omega}^2, \tilde{\delta}^2, E_1, E_2)$ such that $L(\mathcal{M}_1) = L_1$ and $L(\mathcal{M}_2) = L_2$. Consider $\mathcal{M} = (Q, \Sigma, \tilde{R}, Z, \tilde{\omega}, \tilde{\delta}, E_1, E_2)$ as follows: $Q = Q_1 \times Q_2$, $\tilde{R} = \{((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0))\}$, $T^{t_0}(q_0, p_0) =$

$T^{t_0}(q_0) \wedge T^{t_0}(p_0)$, $I^{t_0}(q_0,p_0) = I^{t_0}(q_0) \wedge I^{t_0}(p_0)$, $F^{t_0}(q_0,p_0) = F^{t_0}(q_0) \vee F^{t_0}(p_0)$,

$$\delta_1((q_1,q_2),a,(p_1,p_2)) = \delta_1^1(q_1,a,p_1) \wedge \delta_1^2(q_2,a,p_2),$$
$$\delta_2((q_1,q_2),a,(p_1,p_2)) = \delta_2^1(q_1,a,p_1) \wedge \delta_2^2(q_2,a,p_2),$$
$$\delta_3((q_1,q_2),a,(p_1,p_2)) = \delta_3^1(q_1,a,p_1) \vee \delta_3^2(q_2,a,p_2),$$
$$\omega_1((q_1,q_2),b) = \omega_1^1(q_1,b) \wedge \omega_1^2(q_2,b),$$
$$\omega_2((q_1,q_2),b) = \omega_2^1(q_1,b) \wedge \omega_2^2(q_2,b),$$
$$\omega_3((q_1,q_2),b) = \omega_3^1(q_1,b) \wedge \omega_3^2(q_2,b).$$

Now, we show that $L(\mathcal{M}_1) \cap L(\mathcal{M}_2) = L(\mathcal{M})$. Let $x = a_1...a_n \in L(\mathcal{M}_1) \cap L(\mathcal{M}_2)$. Then $x \in L(\mathcal{M}_1)$ and $x \in L(\mathcal{M}_2)$. So, there exist $q \in Q_1$ and $q \in Q_2$ such that

$$\tilde{\delta}_1^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \wedge \tilde{\omega}_1^1((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) > 0,$$
$$\tilde{\delta}_2^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \wedge \tilde{\omega}_2^1((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) > 0,$$
$$\tilde{\delta}_3^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \vee \tilde{\omega}_3^1((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) < 1$$
$$\tilde{\delta}_1^{*2}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \wedge \tilde{\omega}_1^2((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) > 0,$$
$$\tilde{\delta}_2^{*2}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \wedge \tilde{\omega}_2^2((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) > 0,$$
$$\tilde{\delta}_3^{*2}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),x,q) \vee \tilde{\omega}_3^2((q,T^{t_{n-1}}(q),I^{t_{n-1}}(q),F^{t_{n-1}}(q)),b) < 1.$$

Therefore, there are $q_1,q_2,...,q_{n-1} \in Q_1$ and $p_1,p_2,...,p_{n-1} \in Q_2$ such that

$$\begin{aligned}
&\tilde{\delta}_1^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),a_1,q_1)\\
&\qquad \wedge ... \wedge \tilde{\delta}_1^{*1}((q_{n-1},T^{t_{n-1}}(q_{n-1}),I^{t_{n-1}}(q_{n-1}),F^{t_{n-1}}(q_{n-1})),a_n,q) > 0,\\
&\tilde{\delta}_2^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),a_1,q_1)\\
&\qquad \wedge ... \wedge \tilde{\delta}_2^{*1}((q_{n-1},T^{t_{n-1}}(q_{n-1}),I^{t_{n-1}}(q_{n-1}),F^{t_{n-1}}(q_{n-1})),a_n,q) > 0,\\
&\tilde{\delta}_3^{*1}((q_0,T^{t_0}(q_0),I^{t_0}(q_0),F^{t_0}(q_0)),a_1,q_1)\\
&\qquad \vee ... \vee \tilde{\delta}_3^{*1}((q_{n-1},T^{t_{n-1}}(q_{n-1}),I^{t_{n-1}}(q_{n-1}),F^{t_{n-1}}(q_{n-1})),a_n,q) < 1,\\
&\tilde{\delta}_1^{*2}((p_0,T^{t_0}(p_0),I^{t_0}(p_0),F^{t_0}(p_0)),a_1,p_1)\\
&\qquad \wedge ... \wedge \tilde{\delta}_1^{*2}((p_{n-1},T^{t_{n-1}}(p_{n-1}),I^{t_{n-1}}(p_{n-1}),F^{t_{n-1}}(p_{n-1})),a_n,p) > 0,\\
&\tilde{\delta}_2^{*2}((p_0,T^{t_0}(p_0),I^{t_0}(p_0),F^{t_0}(p_0)),a_1,p_1)\\
&\qquad \wedge ... \wedge \tilde{\delta}_2^{*2}((p_{n-1},T^{t_{n-1}}(p_{n-1}),I^{t_{n-1}}(p_{n-1}),F^{t_{n-1}}(p_{n-1})),a_n,p) > 0,\\
&\tilde{\delta}_3^{*2}((p_0,T^{t_0}(p_0),I^{t_0}(p_0),F^{t_0}(p_0)),a_1,p_1)\\
&\qquad \vee ... \vee \tilde{\delta}_3^{*2}((p_{n-1},T^{t_{n-1}}(p_{n-1}),I^{t_{n-1}}(p_{n-1}),F^{t_{n-1}}(p_{n-1})),a_n,p) < 1.
\end{aligned}$$

Then $T^{t_0}(q_0) > 0$, $I^{t_0}(q_0) > 0$, $F^{t_0}(q_0) < 1$, $T^{t_0}(p_0) > 0$, $I^{t_0}(p_0) > 0$, $F^{t_0}(p_0) < 1$. So, $T^{t_0}(q_0,p_0) > 0$, $I^{t_0}(q_0,p_0) > 0$, $F^{t_0}(q_0,p_0) < 1$. Also, $\delta_1^1(q_0,a_1,q_1) > 0$, $\delta_2^1(q_0,a_1,q_1) > 0$, $\delta_3^1(q_0,a_1,q_1) < 1$, and $\delta_1^2(p_0,a_1,p_1) > 0$, $\delta_2^2(p_0,a_1,p_1) > 0$,

$\delta_3^2(p_0, a_1, p_1) < 1$. Then $\delta_1((q_0, p_0), a_1, (q_1, p_1)) > 0$, $\delta_2((q_0, p_0), a_1, (q_1, p_1)) > 0$, $\delta_3((q_0, p_0), a_1, (q_1, p_1)) < 1$. Similarly, $\delta_1((q_i, p_i), a_{i+1}, (q_{i+1}, p_{i+1})) > 0$, $\delta_2((q_i, p_i), a_{i+1}, (q_{i+1}, p_{i+1})) > 0$, $\delta_3((q_i, p_i), a_{i+1}, (q_{i+1}, p_{i+1})) < 1$, where $i = 1, 2, ..., n-1$ and $q_n = q, p_n = p$. So,

$$\begin{aligned}
&\tilde{\delta}_1^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p)) > 0,\\
&\tilde{\delta}_2^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p)) > 0,\\
&\tilde{\delta}_3^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p)) < 1,
\end{aligned}$$

Also,

$$\begin{aligned}
&\tilde{\omega}_1(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) > 0,\\
&\tilde{\omega}_2(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) > 0,\\
&\tilde{\omega}_3(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) < 1.
\end{aligned}$$

Therefore, $x \in L(\mathcal{M})$ and $L(\mathcal{M}_1) \cap L(\mathcal{M}_2) \subseteq L(\mathcal{M})$. Now, let $x = a_1 a_2 ... a_n \in L(\mathcal{M})$. Then there exists $(q, p) \in Q_1 \times Q_2$ such that

$$\begin{aligned}
&\tilde{\delta}_1^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p))\\
&\qquad \wedge \tilde{\omega}_1(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) > 0,\\
&\tilde{\delta}_2^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p))\\
&\qquad \wedge \tilde{\omega}_2(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) > 0,\\
&\tilde{\delta}_3^*(((q_0, p_0), T^{t_0}(q_0, p_0), I^{t_0}(q_0, p_0), F^{t_0}(q_0, p_0)), x, (q, p))\\
&\qquad \vee \tilde{\omega}_3(((q, p), T^{t_{n-1}}(q, p), I^{t_{n-1}}(q, p), F^{t_{n-1}}(q, p)), b) < 1.
\end{aligned} \tag{9}$$

Since $T^{t_0}(q_0) > 0$, $I^{t_0}(q_0) > 0$, $F^{t_0}(q_0) < 1$, $T^{t_0}(p_0) > 0$, $I^{t_0}(p_0) > 0$, $F^{t_0}(p_0) < 1$, and by considering the definition of δ, we have

$$\begin{aligned}
&\tilde{\delta}_1^{*1}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) > 0,\\
&\tilde{\delta}_2^{*1}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) > 0,\\
&\tilde{\delta}_3^{*1}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) < 1,\\
&\tilde{\delta}_1^{*2}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) > 0,\\
&\tilde{\delta}_2^{*2}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) > 0,\\
&\tilde{\delta}_3^{*2}((q_0, T^{t_0}(q_0), I^{t_0}(q_0), F^{t_0}(q_0)), x, q) < 1,
\end{aligned}$$

and by the definition of ω and (9) we have:

$$\tilde{\omega}_1^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\omega}_2^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\omega}_3^1((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) < 1,$$
$$\tilde{\omega}_1^2((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\omega}_2^2((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) > 0,$$
$$\tilde{\omega}_3^2((q, T^{t_{n-1}}(q), I^{t_{n-1}}(q), F^{t_{n-1}}(q)), b) < 1.$$

Therefore, $x \in L(\mathcal{M}_1)$ and $x \in L(\mathcal{M}_2)$. So, $L(\mathcal{M}) \subseteq L(\mathcal{M}_1) \cap L(\mathcal{M}_2)$. Hence, $L(\mathcal{M}) = L(\mathcal{M}_1) \cap L(\mathcal{M}_2)$. □

5. Conclusion

In this note, by using the notions of single-valued neutrosophic set and general fuzzy automaton, we presented the concepts of single-valued neutrosophic general automaton, complete and deterministic single-valued neutrosophic general automaton. For a given single-valued neutrosophic general machine we were given a minimal single-valued neutrosophic general automaton that preserves the language. After that, we presented the closure properties such as union and intersection for single-valued neutrosophic general automaton.

Now, there is an important question: Is there a relationship between single-valued neutrosophic general automata and finite group and topology? As well, is there any relation between neutrosophic general automata and vector space?

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