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IMPLEMENTATION OF EM ALGORITHM BASED ON NON-PRECISE OBSERVATIONS

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Abstract. The EM algorithm is a powerful tool and generic useful device in a variety of problems for maximum likelihood estimation with incomplete data which usually appears in practice. Here, the term "incomplete" means a general state and in different situations it can mean different meanings, such as lost data, open source data, censored observations, etc. This paper introduces an application of the EM algorithm in which the meaning of "incomplete" data is non-precise or fuzzy observations. The proposed approach in this paper for estimating an unknown parameter in the parametric statistical model by maximizing the likelihood function based on fuzzy observations. Meanwhile, this article presents a case study in the electronics industry, which is an extension of a well-known example used in introductions to the EM algorithm and focuses on the applicability of the algorithm in a fuzzy environment. This paper can be useful for graduate students to understand the subject in fuzzy environment and moreover to use the EM algorithm in more complex examples.

Keywords: EM algorithm, Exponential distribution, Fuzzy Statistics, Fuzzy data, Maximum likelihood estimation.
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1. Preliminaries

1.1. **EM algorithm with crisp data.** In statistics, the Expectation-Maximization (EM) algorithm is a powerful and iterative tool for computing maximum likelihood estimates (MLEs) with incomplete data, introduced in [2]. Incomplete is a general word that, according to the situation, has different meanings such as missing values, unknown components, censored observations and latent variables [4]. For instance, there exist some features that are observable for some cases and not available for others (which we take NaN easily). If we can determine these missing features, our predictions would be way better rather than substituting them with NaNs or mean or some other means. The EM algorithm helps us to infer those hidden variables using the ones that are observable in the dataset which causes to better predictions.

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Suppose that $\mathbf{Y}$ denote the vector of observed data and $\mathbf{X}$ the vector of incomplete data. Moreover, θ is the unknown parameter of interest and $l_c(\theta)$ is the hypothetically complete-data log-likelihood, defined for any θ in the parameter space Θ. The EM algorithm starts with an initial parameter value $\theta^{(0)} \in \Theta$ and repeats two following steps until convergence:

- **E step:** Compute $l_c^{(j)}(\theta) = E_{\mathbf{X}|\mathbf{Y},\theta^{(j-1)}}\left[l_c(\theta)\right]$, where the expectation is taken with respect to the conditional distribution of the complete-data $\mathbf{X}$ (which are not in the hand and not completely observed) given the observed incomplete-data $\mathbf{Y}$. It must be noted that the current numerical value $\theta^{(j-1)}$ is used in evaluating the expected value.
- **M step:** Find $\theta^{(j)} \in \Theta$ that maximizes $l_c^{(j)}(\theta)$.

Iterating for $j = 1, 2, ...$ between the E and M steps leads to a sequence $\theta^{(1)}, \theta^{(2)}, ...$ that converges to a local maximum of the observed-data log-likelihood, if it exists, under fairly general conditions [5].

1.2. **Fuzzy observation from a precise random variable.** The experimenter may be confront with some practically challenges in sampling from a real-world random variable, which cause to approximately record data by non-precise / fuzzy numbers. For instance, several applied situations are mentioned in bellow where the observed data are fuzzy rather than crisp:

(1) The maximum depth of the water in a current river;
(2) Measuring the volume of gas coming from the mouth of a volcano per hour (or from a jet engine per second under some specific conditions);
(3) The amount of interest / satisfaction of a worker from her / his job;
(4) The amount of the life utility for a mission time;
(5) The length of time a worker's interest in job from the start of her / his career;
(6) Foodstuff corruption is gradual. So the length of time of a fridge can keep healthy food / fruit;
(7) The monthly income of a taxi driver;
(8) The time of sunrise / sunset. Therefore, the length of day is also a fuzzy number, which its fuzziness of which is greater in the polar countries (e.g. Norway and Finland) than in the tropical countries;
(9) The lifetime of a battery, and
(10) The tolerance threshold for a patient.

As mentioned earlier, the EM algorithm is a powerful tool for maximum likelihood estimation based on incomplete data, and fuzzy data can be considered as one of the exemplified for incomplete data, since in this case, instead of being precisely record the observation, only an approximate value (in the form of a fuzzy number) is observed and recorded.

The rest of this paper is organized as follows. Some preliminary definitions such as the conditional probability and the conditional mathematical expectation based on fuzzy data are presented in Section 2. The EM algorithm based on fuzzy observations is presented in Section 3. A case study on the lifetime of transistors is presented in Section 4 to estimate the unknown parameter of the exponential distribution in a fuzzy environment. Finally, conclusions are presented in Section 5.

2. Conditional probability based on fuzzy data

In this section, we present some preliminary definitions which are needed for the next section.

Definition 1 The conditional probability density function (p.d.f.) / probability mass function (p.m.f.) of X given the fuzzy observation $\tilde{x}$ is denoted by symbol $f_\theta(x|\ X \in \tilde{x})$ and defined as

$$f_\theta(x|\ X \in \tilde{x}) = \frac{\tilde{x}(x)\ f_\theta(x)}{\int \tilde{x}(x)\ f_\theta(x)\ dx}, \tag{1}$$

where $f_\theta(x)$ is p.d.f. / p.m.f. of X. Replace integration by summation in discrete cases.

It must be mentioned that a similar formula proposed for the density of X based on fuzzy information in [7] from another point of view.

Remark 1 The introduced $f_\theta(x|\ X \in \tilde{x})$ in Definition 1 is a p.d.f. / p.m.f., since $f_\theta(x|\ X \in \tilde{x}) \geq 0, \forall x \in R$ and $\int_{-\infty}^{\infty} f_\theta(x|\ X \in \tilde{x})\ dx = 1$.

Definition 2 Let $\mathbf{X} = (X_1, ..., X_n)$ be a random sample from p.d.f. / p.m.f. $f_\theta(x)$ and also $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n)$ denotes its observation in which $\tilde{x}_i$ is the fuzzy observed number for the random variable X_i with the membership function $\tilde{x}_i(x)$, for $i = 1, ..., n$. As a function of θ, *the likelihood function based on fuzzy observations* $\tilde{\mathbf{x}}$ is defined by

$$\begin{aligned} L(\theta|\ \tilde{\mathbf{x}}) &= L(\theta|\ \tilde{x}_1, ..., \tilde{x}_n) \\ &= \prod_{i=1}^{n} P_\theta\left(X_i \in \tilde{x}_i\right) \\ &= \prod_{i=1}^{n} \int_{-\infty}^{\infty} \tilde{x}_i(x)\ f_\theta(x)\ dx. \end{aligned} \tag{2}$$

Remark 2 Regarding to the Zadeh's probability of fuzzy event [8], the conditional probability of the fuzzy event $\tilde{x}'$ given the fuzzy event $\tilde{x}$ is equal to

$$\begin{aligned} P_\theta\left(X \in \tilde{x}'|\ X \in \tilde{x}\right) &= \int \tilde{x}'(x)\ f_\theta(x|\ X \in \tilde{x})\ dx \\ &= \frac{\int \tilde{x}'(x)\ \tilde{x}(x)\ f_\theta(x)\ dx}{\int \tilde{x}(x)\ f_\theta(x)\ dx} \\ &= \frac{P_\theta\left(X \in (\tilde{x}' \cap \tilde{x})\right)}{P_\theta\left(X \in \tilde{x}\right)}. \end{aligned} \tag{3}$$

Here, the membership function of the union of two fuzzy sets $\tilde{x}$ and $\tilde{x}'$ is defined by $(\tilde{x}' \cap \tilde{x})(x) = \tilde{x}'(x)\ \tilde{x}(x)$.

Remark 3 According to the introduced p.d.f. / p.m.f. in Definition 1, the conditional mathematical expectation of random variable X given the fuzzy observation $\tilde{x}$ is equivalent to

$$\begin{aligned} E_\theta(X|\ X \in \tilde{x}) &= \int_{-\infty}^{\infty} x\ f_\theta(x|\ X \in \tilde{x})\ dx \\ &= \frac{\int x\ \tilde{x}(x)\ f_\theta(x)\ dx}{\int \tilde{x}(x)\ f_\theta(x)\ dx}. \end{aligned} \tag{4}$$

3. EM algorithm with fuzzy data

Suppose that the p.d.f. / p.m.f. of X has an unknown parameter θ and we are going to obtain MLE for θ by the extended version of EM algorithm based on the observed fuzzy sample $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n)$. Now, in order to the maximum likelihood estimation based on the observed fuzzy sample $\tilde{\mathbf{x}}$, we can rewrite two presented EM steps in Section 1.1 as follows:

- **E step:** Compute $l_c^{(j)}(\theta) = E_{\mathbf{X}|\tilde{\mathbf{x}},\theta^{(j-1)}}[l_c(\theta)]$, where the expectation is taken with respect to the conditional distribution of the complete-data $\mathbf{X}$ (which are not in the hand and not completely observed) given the observed incomplete / fuzzy data $\tilde{\mathbf{x}}$.
- **M step:** Find $\theta^{(j)} \in \Theta$ that maximizes $l_c^{(j)}(\theta)$.

Here, all data are assumed to be recorded by fuzzy numbers.

Proposition 1 The maximization of the complete-data log-likelihood $l_c^{(j)}(\theta)$ in j-th iteration of EM algorithm based on the fuzzy sample $\tilde{\mathbf{x}}$ is equivalent to the maximization of

$$\textbf{i)}\quad F_c^{(j)}(\theta) = \sum_{i=1}^{n} \int_{supp(\tilde{x}_i)} \ln(f_\theta(x))\, \tilde{x}_i(x)\ f_{\theta^{(j-1)}}(x)\ dx \tag{5}$$

and also is equivalent to the maximization of

$$\textbf{ii)}\quad G_c^{(j)}(\theta) = \prod_{i=1}^{n} \int_{supp(\tilde{x}_i)} f_\theta(x)\ \tilde{x}_i(x)\ f_{\theta^{(j-1)}}(x)\ dx, \tag{6}$$

where $supp(\tilde{x}_i)$ is the support of the fuzzy number $\tilde{x}_i$.

Proof. Based on fuzzy observations, the complete-data log-likelihood $l_c^{(j)}(\theta)$ in j-th iteration of EM algorithm is equal to

$$\begin{aligned} l_c^{(j)}(\theta) &= E_{\mathbf{X}|\tilde{\mathbf{x}},\theta^{(j-1)}}[\ln\ L_c(\theta)] \\ &= E_{\theta^{(j-1)}}\left[\ln\left(\prod_{i=1}^{n} f_\theta(X_i)\right)\ |\ X_1 \in \tilde{x}_1, ..., X_n \in \tilde{x}_n\right] \end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{n} E_{\theta^{(j-1)}}\left[\ln\left(f_\theta(X_i)\right) \mid X_i \in \tilde{x}_i\right], \qquad \text{since } X_i\text{'s are i.i.d.}\\
&= \sum_{i=1}^{n} \int_{-\infty}^{\infty} \ln\left(f_\theta(x_i)\right) f_{\theta^{(j-1)}}(x_i|X_i \in \tilde{x}_i)\, dx_i, \qquad \text{by Remark 3}\\
&= \sum_{i=1}^{n} \frac{\int_{supp(\tilde{x}_i)} \ln\left(f_\theta(x_i)\right) \tilde{x}_i(x_i)\, f_{\theta^{(j-1)}}(x_i)\, dx_i}{\int_{supp(\tilde{x}_i)} \tilde{x}_i(x_i)\, f_{\theta^{(j-1)}}(x_i)\, dx_i}, \qquad \text{by Definition 1}\\
&\propto \sum_{i=1}^{n} \int_{supp(\tilde{x}_i)} \ln\left(f_\theta(x_i)\right) \tilde{x}_i(x_i)\, f_{\theta^{(j-1)}}(x_i)\, dx_i, \qquad \forall\, \theta \in \Theta, \qquad (7)
\end{aligned}
$$

since the denominator in Relation (7) is a fix and positive real number, where the symbol $\propto$ is the proportion. Similarly, the proof of (ii) is as follows

$$
\begin{aligned}
l_c^{(j)}(\theta) &\propto E_{\mathbf{X}|\tilde{\mathbf{x}},\theta^{(j-1)}}\left[L_c(\theta)\right]\\
&= E_{\theta^{(j-1)}}\left[\prod_{i=1}^{n} f_\theta(X_i) \mid X_1 \in \tilde{x}_1, ..., X_n \in \tilde{x}_n\right]\\
&= \prod_{i=1}^{n} E_{\theta^{(j-1)}}\left[f_\theta(X_i) \mid X_i \in \tilde{x}_i\right],\\
&= \prod_{i=1}^{n} \int_{-\infty}^{\infty} f_\theta(x_i)\; f_{\theta^{(j-1)}}(x_i|X_i \in \tilde{x}_i)\, dx_i,\\
&= \prod_{i=1}^{n} \frac{\int_{supp(\tilde{x}_i)} f_\theta(x_i)\; \tilde{x}_i(x_i)\; f_{\theta^{(j-1)}}(x_i)\, dx_i}{\int_{supp(\tilde{x}_i)} \tilde{x}_i(x_i)\; f_{\theta^{(j-1)}}(x_i)\, dx_i},\\
&\propto \prod_{i=1}^{n} \int_{supp(\tilde{x}_i)} f_\theta(x_i)\; \tilde{x}_i(x_i)\; f_{\theta^{(j-1)}}(x_i)\, dx_i, \qquad \forall\, \theta \in \Theta. \qquad (8)
\end{aligned}
$$

Remark 4 In the presented EM algorithm with fuzzy observation, one can numerically maximize $F_c^{(j)}(\theta)$ or $G_c^{(j)}(\theta)$ rather than the theoretically maximization of $l_c^{(j)}(\theta)$.

A few applications of EM algorithm with fuzzy data, which was first proposed in [3], exist in literature. Hence, we intend to present a case study for the lifetime of a particular type of transistor based on exponential distribution in the next section.

4. Case study in applying theories to practice

Althought the proposed EM algorithm is general and satisfy for any fuzzy number, but because of the simplicity, here we use from the triangular fuzzy numbers with the symbol $\tilde{x} = T(c, s_l, s_r)$ and the membership function

$$
\tilde{x}(x) = \begin{cases} \frac{x-c+s_l}{s_l} & \text{if} \quad c - s_l \le x < c \\ \frac{c+s_r-x}{s_r} & \text{if} \quad c \le x < c + s_r \\ 0 & \text{elsewhere} \end{cases} \qquad (9)
$$

in which c, s_l and s_r are the core, the left spread and the right spread of the fuzzy number $\tilde{x}$ (see Figure 1).

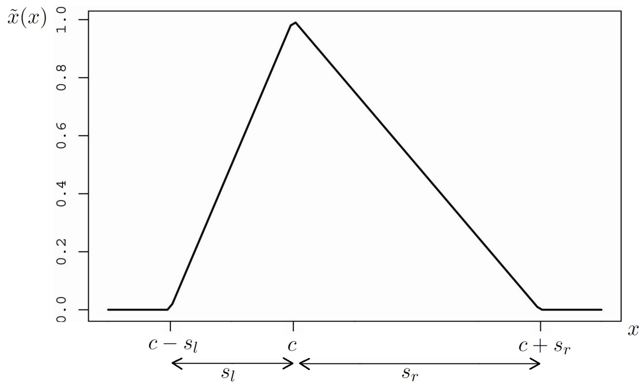

FIGURE 1. The membership function of triangular fuzzy number $\tilde{x} = T(c, s_l, s_r)$.

4.1. **Theoretical perspective.** Suppose $\mathbf{X} = (X_1, ..., X_n)$ is a random sample from the exponential distribution with the unknown parameter λ. Let all the observations are fuzzy and therefore, instead of observing the crisp value of x_i, a membership function for fuzzy data $\tilde{x}_i$ has been recorded for $i = 1, ..., n$. In other words, we wish to find the MLE of the unknown parameter λ based on fuzzy-valued observations $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n)$ by the EM algorithm. The likelihood function based on these fuzzy data is equal to

$$L(\lambda|\tilde{\mathbf{x}}) = \prod_{i=1}^{n} P_\lambda(X_i \in \tilde{x}_i) = \prod_{i=1}^{n} \int_0^\infty \tilde{x}_i(x)\ f_\lambda(x)\ dx. \tag{10}$$

Although to find $MLE(\lambda)$ it is not possible to derivate from this function (or equivalently from its logarithm) with respect to λ, but it is possible to maximize it using the numerical methods (such as the Newton Raphson). In order to use the EM algorithm, which is the proposed approach in this paper, we first take the logarithm of likelihood function based on the completed / real-valued data, i.e.

$$L_c(\lambda) = \lambda^n \exp(-\lambda \sum_{i=1}^{n} x_i), \tag{11}$$

or equivalently,

$$l_c(\lambda) = n\ ln(\lambda) - \lambda \sum_{i=1}^{n} x_i, \tag{12}$$

and then $MLE(\lambda) = \frac{1}{\bar{x}} = \frac{n}{\sum_{i=1}^{n} x_i}$. It should be noted that the purpose is computing MLE based on fuzzy (and not crisp) observation. By Definition 1, the conditional p.d.f. of X_i given the fuzzy observation $\tilde{x}_i$ is

$$\begin{aligned} f_\lambda(x_i|\ X_i \in \tilde{x}_i) &= \frac{\tilde{x}_i(x_i)\ f_\lambda(x_i)}{\int \tilde{x}_i(x)\ f_\lambda(x_i)\ dx_i} \\ &= \frac{\tilde{x}_i(x_i)\ \lambda\ \exp(-\lambda\ x_i)}{\int \tilde{x}_i(x)\ \lambda\ \exp(-\lambda\ x)\ dx} \\ &= \frac{\tilde{x}_i(x_i)\ \exp(-\lambda\ x_i)}{\int \tilde{x}_i(x)\ \exp(-\lambda\ x)\ dx}, \qquad i = 1, ..., n. \end{aligned} \tag{13}$$

According to the presented algorithm in Section 3, here $\tilde{\mathbf{x}} = (\tilde{x}_1, ..., \tilde{x}_n)$ is the incomplete observed data and $\mathbf{x} = (x_1, ..., x_n)$ is the complete data which are not in the hand. Therefore, the logarithm of likelihood function based on the fuzzy data (E step) in i-th iteration of algorithm is equal to

$$\begin{aligned} l_c^{(j)}(\lambda) &= E_{\mathbf{X}|\tilde{\mathbf{x}},\lambda^{(j-1)}}\left[l_c(\lambda)\right] \\ &= E_{\lambda^{(j-1)}}\left[l_c(\lambda)|\ \ X_1 \in \tilde{x}_1, ...,\ X_n \in \tilde{x}_n\right] \\ &= n\ ln(\lambda) - \lambda \sum_{i=1}^{n} E_{\lambda^{(j-1)}}(X_i|\ \ X_1 \in \tilde{x}_1, ...,\ X_n \in \tilde{x}_n) \\ &= n\ ln(\lambda) - \lambda \sum_{i=1}^{n} E_{\lambda^{(j-1)}}(X_i|\ \ X_i \in \tilde{x}_i). \end{aligned} \tag{14}$$

By assuming constant value of $\lambda^{(j-1)}$ (because its value is set in the previous iteration of the algorithm), the maximum of Eq. (14) can be calculated via equation $\frac{\partial l_c^{(j)}(\lambda)}{\partial \lambda} = 0$ as follows (M step)

$$\hat{\lambda}^{(j)} = \frac{n}{\sum_{i=1}^{n} E_{\lambda^{(j-1)}}(X_i|\ X_i \in \tilde{x}_i)}. \tag{15}$$

From Remark 3, we have

$$\begin{aligned} E_{\lambda^{(j-1)}}(X_i|\ X_i \in \tilde{x}_i) &= \int_{-\infty}^{\infty} x\ f_{\lambda^{(j-1)}}(x|\ X_i \in \tilde{x}_i)\ dx \\ &= \frac{\int_0^\infty x\ \ \tilde{x}_i(x)\ f_{\lambda^{(j-1)}}(x)\ dx}{\int_0^\infty \tilde{x}_i(x)\ f_{\lambda^{(j-1)}}(x)\ dx} \\ &= \left\{ \frac{\int\ x\ \ \tilde{x}_i(x)\ \exp(-\lambda\ x)\ dx}{\int \tilde{x}_i(x)\ \exp(-\lambda\ x)\ dx} \right\}_{\lambda=\lambda^{(j-1)}}. \end{aligned} \tag{16}$$

4.2. **Practical perspective.** A particular type of transistor operates at a high efficiency for a period of time and then continues its duties at a lower, but acceptable, quality for another period time. Then, it completely loses its efficiency and burns. In other words, the burning / destruction of such particular type of transistors occur gradually over a period of operating time. Therefore, the manager decides to record the observed lifetimes of them by triangular fuzzy numbers.

Let the lifetime of transistors is distributed exponentially with an unknown parameter λ [1]. Due to the high price of this type of transistor, a random sample of five transistors is considered and their lifetimes are recorded by triangular fuzzy numbers (in terms of years) to estimate the unknown parameter λ in the exponential distribution (see Figure 2)

$$\tilde{x}_1 = T(1.019, 0.865, 1.374)$$
$$\tilde{x}_2 = T(1.141, 0.316, 1.548)$$
$$\tilde{x}_3 = T(0.806, 0.523, 0.944)$$
$$\tilde{x}_4 = T(0.679, 0.292, 0.428)$$
$$\tilde{x}_5 = T(0.536, 0.038, 1.274)$$

Table 1 shows the result of the algorithm iteration and its convergence based on these fuzzy data, by two different initial parameter values $\lambda_1^{(0)} = 5$ and $\lambda_2^{(0)} = 30$, for which both initial values, the EM algorithm has reached a similar convergence ($\hat{\lambda} = 1.042$). Moreover, the estimated probability density function for the lifetime of transistors is shown in Figure 2 by dash-dotted line.

Table 1. The convergence process of the EM algorithm for two initial values

The number of iteration (j)	$\lambda_1^{(j)}$	$\lambda_2^{(j)}$
0	5	30
1	1.397	2.031
2	1.079	1.143
3	1.045	1.052
4	1.042	1.043
5	1.042	1.042
6		1.042

5. Conclusions

The EM algorithm is one of the numerical methods to calculate the MLE based on incomplete data. The present work propose a new application of EM algorithm, in which the incomplete data are non-precise / fuzzy. It must be emphasized that Denoeux [3] showed for the first time that the EM algorithm can be adapted to handle estimation problems based on fuzzy data. His method is general and covers various situations (where some observations are crisp and the others are fuzzy) and, at the same time, is complex for the computation and formulation. But in this paper, we have tried to consider a special case of [3] in which all observations are fuzzy, since we believe that this special case can be simpler and leads for the graduate students to understanding the

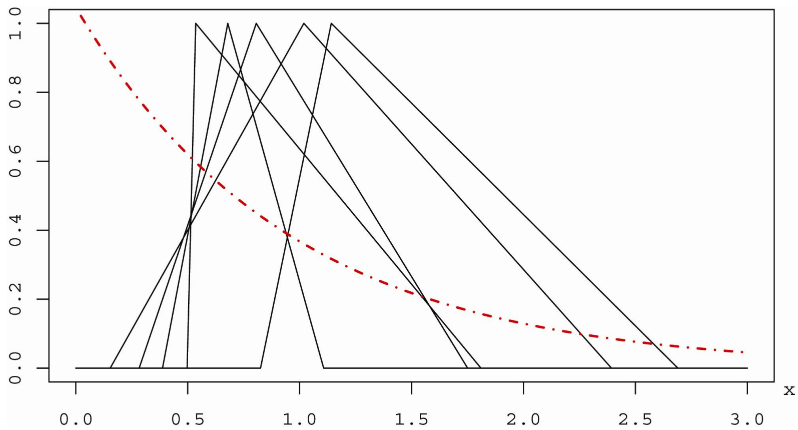

Figure 2. The membership functions of triangular fuzzy observations and the estimated probability density function for the lifetime of transistors.

approach and applying the EM algorithm in a fuzzy environment. From the applied point of view, the *R* software package *EM.Fuzzy* can be useful for the practitioners to find the MLEs by the EM algorithm on the basis of fuzzy information. Moreover, this package contains various statistical examples for the EM algorithm based on the fuzzy simulated data.

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