

## ON THE ANNIHILATORS OF GENERALIZED LOCAL COHOMOLOGY MODULES

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**ABSTRACT.** Let  $\mathfrak{a}$  be an ideal of Noetherian ring  $R$  and  $M, N$  be two finitely generated  $R$ -modules. In this paper, we obtain some results about the annihilators of top generalized local cohomology modules. We define  $T := T_R(\mathfrak{a}, M, N)$  as the largest submodule of  $N$  such that  $\text{cd}(\mathfrak{a}, M, T_R(\mathfrak{a}, M, N)) < \text{cd}(\mathfrak{a}, M, N)$ . Let  $(R, \mathfrak{m})$  be a complete Gorenstein Noetherian local ring such that  $\text{pd}(M) = d < \infty$ ,  $\dim N = \dim R = n < \infty$  and  $\text{cd}(\mathfrak{a}, M, N) = d+n$ . We prove that if  $\text{Ass}_R(\text{Ext}_R^d(M, N/T)) \subseteq \text{Ass}_R R$ , then  $\text{Ann}_R(H_{\mathfrak{m}}^{d+n}(M, N)) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T)$ .

*Keywords:* Annihilator, Local cohomology, Noetherian ring.

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### 1. Introduction

Throughout this paper,  $R$  is a commutative Noetherian ring with identity,  $\mathfrak{a}$  is an ideal of  $R$ ,  $M$  and  $N$  are two  $R$ -modules.

For each  $i \geq 0$ , the  $i$ -th local cohomology module  $M$  with respect to an ideal  $\mathfrak{a}$  is defined as  $H_{\mathfrak{a}}^i(M) = \varinjlim_n \text{Ext}_R^i(R/\mathfrak{a}^n, M)$ . For basic results about local cohomology we refer the reader to [6]. A Generalization of local cohomology functor has been given by J. Herzog in [10]. The  $i$ -th generalized local cohomology module of  $M$  and  $N$  with respect to  $\mathfrak{a}$  is denoted by  $H_{\mathfrak{a}}^i(M, N) := \varinjlim_n \text{Ext}_R^i(M/\mathfrak{a}^n M, N)$ .

An important problem concerning local cohomology is determining the annihilators of the  $i$ -th local cohomology module  $H_{\mathfrak{a}}^i(M)$ . This problem has been studied by several authors, see for example [2], [3], [4], [5], [9] and [11]. In [5], Bahmanpour et al. proved an interesting result about the annihilator  $\text{Ann}_R(H_{\mathfrak{m}}^{\dim(M)}(M))$  in the case  $(R, \mathfrak{m})$  is a complete local ring. In fact, in [5, Theorem 2.6], they proved that if  $(R, \mathfrak{m})$  is a complete local ring then  $\text{Ann}_R(H_{\mathfrak{m}}^{\dim(M)}(M)) = \text{Ann}_R(M/T_R(M))$  where  $T_R(M) = \cup\{N : N \leq M \text{ and } \dim N < \dim M\}$ .

Atazadeh et al. in [2] generalized this main result by determining  $\text{Ann}_R(H_{\mathfrak{a}}^{\dim(M)}(M))$  for an arbitrary Noetherian ring  $R$ . In [2, Theorem 2.3],

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by using the above main result, they showed that, if  $H_{\mathfrak{a}}^{\dim(M)}(M) \neq 0$ , then  $\text{Ann}_R(H_{\mathfrak{a}}^{\dim(M)}(M)) = \text{Ann}_R(M/T_R(\mathfrak{a}, M))$  where  $T_R(\mathfrak{a}, M) = \cup\{N : N \leq M \text{ and } \text{cd}(\mathfrak{a}, N) < \text{cd}(\mathfrak{a}, M)\}$ .

Not so much is known about the annihilators of top generalized local cohomology modules. Here, we obtain some results about them. Let  $\mathfrak{a}$  be an ideal of Noetherian ring  $R$  and  $M, N$  two finitely generated  $R$ -modules. At first, we define  $T_R(\mathfrak{a}, M, N)$  as the largest submodule of  $N$  such that  $\text{cd}(\mathfrak{a}, M, T_R(\mathfrak{a}, M, N)) < \text{cd}(\mathfrak{a}, M, N)$ . Then we prove the following main result:

**Theorem 1.1.** *Let  $(R, \mathfrak{m})$  be a complete Gorenstein Noetherian local ring. Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules and  $T := T_R(\mathfrak{m}, M, N)$ . Assume that  $\text{pd}(M) = d < \infty$ ,  $\dim N = \dim R = n < \infty$  and  $\text{cd}(\mathfrak{m}, M, N) = d+n$ . Also, assume that  $\text{Ass}_R(\text{Ext}_R^d(M, N/T)) \subseteq \text{Ass}_R R$ . Then  $\text{Ann}_R(H_{\mathfrak{m}}^{d+n}(M, N)) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T)$ .*

In [13, Theorem 3.3], the author has determined the annihilators of generalized local cohomology modules. In Remark 2.9 we show that his claimed proof is invalid.

## 2. Main Results

Recall that, for any two  $R$ -modules  $M$  and  $N$ , the cohomological dimension of  $M$  and  $N$  with respect to an ideal  $\mathfrak{a}$  of a commutative Noetherian ring  $R$ , is defined as

$$\text{cd}(\mathfrak{a}, M, N) = \sup\{i \in \mathbb{N}_0 : H_{\mathfrak{a}}^i(M, N) \neq 0\}.$$

For more details see [1].

We need the following result in our proofs.

**Lemma 2.1.** *Let  $R$  be a Noetherian ring and  $\mathfrak{a}$  be an ideal of  $R$ . Let  $M, N, L$  be finitely generated  $R$ -modules such that  $\text{Supp } N \subseteq \text{Supp } L$ . Then,  $\text{cd}(\mathfrak{a}, M, N) \leq \text{cd}(\mathfrak{a}, M, L)$ .*

**Proof.** See [1, Theorem B].  $\square$

We give the following definition which is a generalization of [3, Definition 2.4].

**Definition 2.2.** Let  $\mathfrak{a}$  be an ideal of Noetherian ring  $R$ ,  $M$  and  $N$  be non-zero finitely generated  $R$ -modules. We denote by  $T_R(\mathfrak{a}, M, N)$  the largest submodule of  $N$  such that  $\text{cd}(\mathfrak{a}, M, T_R(\mathfrak{a}, M, N)) < \text{cd}(\mathfrak{a}, M, N)$ .

**Proposition 2.3.** *Let  $R$  be a Noetherian ring and  $\mathfrak{a}$  be an ideal of  $R$ . Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules. Assume that  $\text{pd}(M) = d < \infty$  and  $\dim N = n < \infty$ . Then  $H_{\mathfrak{a}}^i(M, N) = 0$  for all  $i > d + n$ .*

**Proof.** It follows by [14, Theorem 3.7].  $\square$

**Lemma 2.4.** *Let  $R$  be a Noetherian ring and  $\mathfrak{a}$  be an ideal of  $R$ . Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules. Assume that  $\text{pd}(M) = d < \infty$ ,*

$\dim N = n < \infty$  and  $\text{cd}(\mathfrak{a}, M, N) = d + n$ . Assume that  $G := N/T_R(\mathfrak{a}, M, N)$ .

Then

- i)  $H_{\mathfrak{a}}^{d+n}(M, G) \cong H_{\mathfrak{a}}^{d+n}(M, N)$  and so  $\text{cd}(\mathfrak{a}, M, G) = d + n$ ,
- ii)  $G$  has no non-zero submodule  $K$  such that  $\text{cd}(\mathfrak{a}, M, K) < \text{cd}(\mathfrak{a}, M, G) = d + n$ ,
- iii)  $\text{Ass } G = \{\mathfrak{p} \in \text{Supp}_R G \mid \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n\} = \{\mathfrak{p} \in \text{Ass}_R N \mid \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n\} \subseteq \{\mathfrak{p} \in \text{Spec } R \mid \dim R/\mathfrak{p} = n\}$ ,
- iv)  $\dim G = n$ ,
- v)  $G$  has no non-zero submodule of dimension less than  $n$ .

**Proof.** i) Set  $T := T_R(\mathfrak{a}, M, N)$ . By using the exact sequence  $0 \rightarrow T \rightarrow N \rightarrow G \rightarrow 0$  we obtain

$$\cdots \rightarrow H_{\mathfrak{a}}^{d+n}(M, T) \rightarrow H_{\mathfrak{a}}^{d+n}(M, N) \rightarrow H_{\mathfrak{a}}^{d+n}(M, G) \rightarrow 0.$$

By Definition 2.2,  $\text{cd}(\mathfrak{a}, M, T) < d + n$  and so  $H_{\mathfrak{a}}^{d+n}(M, T) = 0$ . Thus the above exact sequence implies that  $H_{\mathfrak{a}}^{d+n}(M, G) \cong H_{\mathfrak{a}}^{d+n}(M, N)$ .

ii) Assume that, there exists a non-zero submodule  $K := U/T$  of  $G = N/T$  such that  $\text{cd}(\mathfrak{a}, M, K) < \text{cd}(\mathfrak{a}, M, G)$ . By Definition 2.2 and (i) we have  $\text{cd}(\mathfrak{a}, M, T) < \text{cd}(\mathfrak{a}, M, N) = \text{cd}(\mathfrak{a}, M, G)$ . On the other hand, from  $0 \rightarrow T \rightarrow U \rightarrow K \rightarrow 0$  and by [1, Theorem A] we have  $\text{cd}(\mathfrak{a}, M, U) = \max\{\text{cd}(\mathfrak{a}, M, T), \text{cd}(\mathfrak{a}, M, K)\}$ . Thus it follows that  $\text{cd}(\mathfrak{a}, M, U) < \text{cd}(\mathfrak{a}, M, G) = \text{cd}(\mathfrak{a}, M, N)$ . But  $T \subsetneq U$  and this is a contradiction by definition of  $T$ .

iii) By (ii)  $G$  has no non-zero submodule  $K$  such that  $\text{cd}(\mathfrak{a}, M, K) < \text{cd}(\mathfrak{a}, M, G) = d + n$ . Thus

$$\text{Ass } G \subseteq \{\mathfrak{p} \in \text{Supp } G : \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n\}.$$

Let  $\mathfrak{p} \in \text{Supp } G$  and  $\text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n$ . Since  $d + n = \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) \leq d + \dim R/\mathfrak{p}$  and  $d + \dim R/\mathfrak{p} \leq d + \dim N = d + n$  we conclude that  $\dim R/\mathfrak{p} = n$ . Since  $\mathfrak{p} \in \text{Supp } G \subseteq \text{Supp } N$  and  $\dim R/\mathfrak{p} = n$  it follows that  $\mathfrak{p} \in \text{Ass } N$  and so

$$\text{Ass } G \subseteq \{\mathfrak{p} \in \text{Supp } G : \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n\} \subseteq \{\mathfrak{p} \in \text{Ass } N : \text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n\}.$$

Now assume that  $\mathfrak{p} \in \text{Ass } N$  and  $\text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) = d + n$ . If  $\mathfrak{p} \in \text{Supp } T$ , then  $\text{cd}(\mathfrak{a}, M, R/\mathfrak{p}) \leq \text{cd}(\mathfrak{a}, M, T) < d + n$  by Lemma 2.1 which is a contradiction. Thus  $\mathfrak{p} \in \text{Supp } G$  and by the above argument  $\dim R/\mathfrak{p} = n$ . Therefore  $\mathfrak{p} \in \text{Ass } G$ .

iv) By (iii) if  $\mathfrak{p} \in \text{Ass}_R G$ , then  $\dim R/\mathfrak{p} = n$ . Thus  $\dim G = n$ .

v) Let  $U$  be a non-zero submodule of  $G$ . By (ii) we have  $\text{cd}(\mathfrak{a}, M, U) = d + n$ . Now if  $\dim U < n$ , then  $H_{\mathfrak{a}}^{d+n}(M, U) = 0$  which is a contradiction. Thus  $\dim U = n$ , as required.  $\square$

**Proposition 2.5.** *Let  $R$  be a Noetherian ring and  $\mathfrak{a}$  be an ideal of  $R$ . Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules. Assume that  $\text{pd}(M) = d < \infty$  and  $\dim N = n < \infty$ . Then  $H_{\mathfrak{a}}^{d+n}(M, N)$  is Artinian and  $H_{\mathfrak{a}}^{d+n}(M, N) \cong \text{Ext}_R^d(M, H_{\mathfrak{a}}^n(N))$ .*

**Proof.** By [8, Proposition 2.2].  $\square$

**Proposition 2.6.** *Let  $(R, \mathfrak{m})$  be a Cohen-Macaulay Noetherian local ring and for every  $\mathfrak{p} \in \text{Ass}_R R$  the zero-dimensional local ring  $R_{\mathfrak{p}}$  is Gorenstein. Let  $M$  be a finitely generated  $R$ -module such that  $\text{Ass}_R M \subseteq \text{Ass}_R R$ . Then there is an exact sequence  $0 \rightarrow M \rightarrow \bigoplus_{i=1}^n R$  for some positive integer  $n$ .*

**Proof.** By [7, Theorem 3.5].  $\square$

In the following, we prove the main result of this paper.

**Theorem 2.7.** *Let  $(R, \mathfrak{m})$  be a complete Gorenstein Noetherian local ring. Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules and  $T := T_R(\mathfrak{m}, M, N)$ . Assume that  $\text{pd}(M) = d < \infty$ ,  $\dim N = \dim R = n < \infty$  and  $\text{cd}(\mathfrak{m}, M, N) = d+n$ . Also, assume that  $\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \text{Ass}_R R$ . Then  $\text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T)$ .*

**Proof.** By Lemma 2.4 (i),  $H_{\mathfrak{m}}^{d+n}(M, N) \cong H_{\mathfrak{m}}^{d+n}(M, N/T)$ . Thus we must show that  $\text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N/T) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T)$ . By Proposition 2.6 there exists a monomorphism

$$f : \text{Ext}_R^d(M, N/T) \rightarrow \bigoplus_{i=1}^t R,$$

for some positive integer  $t$ . We claim

$$\text{Ann}_R(\text{Hom}_R(\text{Ext}_R^d(M, N/T), \bigoplus_{i=1}^t R) = \text{Ann}_R \text{Ext}_R^d(M, N/T).$$

If  $r \in \text{Ann}_R(\text{Hom}_R(\text{Ext}_R^d(M, N/T), \bigoplus_{i=1}^t R))$ , then  $rf = 0$ . Thus  $f(r \text{Ext}_R^d(M, N/T)) = 0$ . Since  $f$  is a monomorphism, it follows that  $r \text{Ext}_R^d(M, N/T) = 0$  and so  $r \in \text{Ann}_R \text{Ext}_R^d(M, N/T)$ . On the other hand, since  $\text{Ext}_R^d(M, -)$  is a right exact functor we have  $\text{Ext}_R^d(M, N/T) \simeq \text{Ext}_R^d(M, R) \otimes N/T$ . Thus by using Proposition 2.5 we have

$$\begin{aligned} & \text{Hom}_R(H_{\mathfrak{m}}^{d+n}(M, N/T), \bigoplus_{i=1}^t E(R/\mathfrak{m})) \\ & \cong \text{Hom}_R(\text{Ext}_R^d(M, H_{\mathfrak{m}}^n(N/T)), \bigoplus_{i=1}^t E(R/\mathfrak{m})) \\ & \cong \text{Hom}_R(\text{Ext}_R^d(M, R) \otimes H_{\mathfrak{m}}^n(N/T), \bigoplus_{i=1}^t E(R/\mathfrak{m})) \\ & \cong \text{Hom}_R(\text{Ext}_R^d(M, R) \otimes H_{\mathfrak{m}}^n(R) \otimes N/T, \bigoplus_{i=1}^t E(R/\mathfrak{m})) \\ & \cong \text{Hom}_R(\text{Ext}_R^d(M, R) \otimes N/T, \text{Hom}_R(H_{\mathfrak{m}}^n(R), \bigoplus_{i=1}^t E(R/\mathfrak{m}))) \\ & \cong \text{Hom}_R(\text{Ext}_R^d(M, N/T), \text{Hom}_R(H_{\mathfrak{m}}^n(R), \bigoplus_{i=1}^t E(R/\mathfrak{m}))). \end{aligned}$$

Since  $R$  is a Gorenstein ring by [6, Lemma 11.2.3] it follows that  $H_{\mathfrak{m}}^n(R) \cong E(R/\mathfrak{m})$  and since  $R$  is complete by [6, Theorem 10.2.11] we conclude that

$$\text{Hom}_R(H_{\mathfrak{m}}^n(R), \bigoplus_{i=1}^t E(R/\mathfrak{m})) \cong \bigoplus_{i=1}^t R.$$

Thus we have

$$\text{Hom}_R(H_{\mathfrak{m}}^{d+n}(M, N/T), \bigoplus_{i=1}^t E(R/\mathfrak{m})) \cong \text{Hom}_R(\text{Ext}_R^d(M, N/T), \bigoplus_{i=1}^t R).$$

Therefore by Lemma 2.4 (i) we have

$$\begin{aligned} \text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N) &= \text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N/T) \\ &\subseteq \text{Ann}_R(\text{Hom}_R(H_{\mathfrak{m}}^{d+n}(M, N/T), \bigoplus_{i=1}^t E(R/\mathfrak{m}))) \end{aligned}$$

$= \text{Ann}_R(\text{Hom}_R(\text{Ext}_R^d(M, N/T), \oplus_{i=1}^t R)) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T)$   
and the proof is complete.  $\square$

**Corollary 2.8.** *Let  $(R, \mathfrak{m})$  be a complete Gorenstein Noetherian local ring. Let  $M$  and  $N$  be non-zero finitely generated  $R$ -modules and  $T := T_R(\mathfrak{m}, M, N)$ . Assume that  $\text{pd}(M) = d < \infty$ ,  $\dim N = \dim R = n < \infty$  and  $\text{cd}(\mathfrak{m}, M, N) = d + n$ . Then*

i) *If  $\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \text{Ass}_R R$ , then*

$$\sqrt{\text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N)} \subseteq \cap_{\mathfrak{p} \in \text{Ass}_R \text{Ext}_R^d(M, N/T)} \mathfrak{p}.$$

ii) *If  $\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \{\mathfrak{p} \in \text{Spec } R \mid \dim R/\mathfrak{p} = n\}$ , then*

$$\text{Ann}_R H_{\mathfrak{m}}^{d+n}(M, N) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T).$$

iii) *If  $\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \text{Ass}_R N/T$ , then*

$$\text{Ann}_R H_{\mathfrak{m}}^{n+d}(M, N) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T).$$

iv) *Let  $\text{Ext}_R^d(M, N/T)$  be a Cohen-Macaulay  $R$ -module of dimension  $n$ . Then*

$$\text{Ann}_R H_{\mathfrak{m}}^{n+d}(M, N) \subseteq \text{Ann}_R \text{Ext}_R^d(M, N/T).$$

v) *Assume that  $M$  is projective. Then*

$$\text{Ann}_R H_{\mathfrak{m}}^n(M, N) \subseteq \text{Ann}_R \text{Hom}_R(M, N/T).$$

**Proof.** i) The result follows by Theorem 2.7.

ii) Since  $\{\mathfrak{p} \in \text{Spec } R \mid \dim R/\mathfrak{p} = n\} \subseteq \text{Ass}_R R$ , the result follows by Theorem 2.7.

iii) By assumption and Lemma 2.4 (iii) we conclude that

$$\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \{\mathfrak{p} \in \text{Spec } R \mid \dim R/\mathfrak{p} = n\},$$

and so the result follows by (ii).

iv) By assumption and [12, Theorem 17.3 (i)] we conclude that  $\text{Ass}_R \text{Ext}_R^d(M, N/T) \subseteq \{\mathfrak{p} \in \text{Spec } R \mid \dim R/\mathfrak{p} = n\}$ . Thus the result follows by (ii).

v) By assumption  $d = \text{pd } M = 0$ . We have

$$\text{Ass}_R \text{Hom}_R(M, N/T) = \text{Supp}_R M \cap \text{Ass}_R N/T \subseteq \text{Ass}_R N/T.$$

Thus Part (iii) completes the proof.  $\square$

*Remark 2.9.* In [13, Theorem 3.3], the author has determined the annihilators of generalized local cohomology modules. But he has made an error in the proof. In fact, for a system of ideals  $\Phi$  in a Noetherian ring  $R$  and two non-zero finitely generated  $R$ -modules  $M, N$  such that  $c := \text{cd}_{\Phi}(M, N)$ , when one apply the functor  $H_{\Phi}(M, -)$  to the map  $N \rightarrow xN$  where  $x \in R$ , the result will not be multiplication by  $x$ . But applyig the local cohomology functor to  $N \rightarrow N$  which is multiplication by  $x$  will be again multiplication by  $x$ .

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